SIMON FRASER UNIVERSITY Department of Economics

Econ 808 Macroeconomic Theory Prof. Kasa Fall 2010

MIDTERM EXAM - October 29 (Solutions)

Answer the following questions True, False, or Uncertain. Briefly explain your answers. No credit without explanation. (10 points each).

1. Non-ergodic Markov processes do not have stationary distributions.

FALSE. A non-ergodic Markov process has more than one stationary distribution.

2. In the Mortensen-Pissarides model, positive productivity shocks increase unemployment.

FALSE. A positive productivity shock shifts up both the Job Creation Curve and the Wage Curve. However, the Wage Curve shifts up less than the Job Creation Curve, since with Nash Bargaining wages rise by only a fraction of the increase in productivity (holding market tightness constant). As a result, there are now profits from opening vacancies. Market tightness increases until profits are driven back down to zero. Since the Beveridge Curve has not changed, a tighter labor market implies equilibrium unemployment falls.

3. According to Ljungqvist and Sargent, relatively high European unemployment rates are caused by relatively generous unemployment compensation policies.

FALSE/UNCERTAIN. Ljungqvist and Sargent argue that it is the <u>interaction</u> of shocks and institutions that caused European unemployment rates to rise during the 1980s. Generous unemployment compensation policies by themselves cannot explain higher unemployment rates, since Europe has always had relatively generous unemployment policies, yet their unemployment rates were relatively <u>low</u> during the 1960s and 70s. L& S argue that labor market 'turbulence' increased during the 1980s, which took the form of an increased risk of human capital loss during job separations. With UI pegged to prior labor market earnings, it then becomes very difficult to find acceptable employment. L& S further argue that human capital erodes while workers are out of work, which makes it even more unlikely that the worker will find acceptable employment. Eventually, with endogenous/costly search, the worker gives up hope and becomes permanently unemployed. This matches the observation that most of the increased unemployment in Europe took the form of a reduced probability of leaving the unemployment pool, rather than an increase in the inflow rate.

The following questions are short answer. Be sure to explain and interpret your answer. Clarity and conciseness will be rewarded.

4. (20 points). Learning to Enjoy Spare Time. A worker's period utility, $u(c_{1t}, c_{2t})$, depends on the amount of market-goods consumed, c_{1t} , and the amount of home-produced goods, c_{2t} (e.g., leisure, recreation). To acquire market-produced goods, the worker must allocate some time, l_{1t} , to market activities that pay a salary of w_t (measured in units of market consumption). There is no borrowing or lending. The market wage is known to evolve according to the process, $w_{t+1} = h(w_t)$.

The quantity of home-produced goods depends on the stock "expertise" that the worker has at the beginning of each period, denoted by a_t , so that $c_{2t} = f(a_t)$. The stock of expertise depreciates at the rate δ , but can be augmented by allocating time to nonmarket activities. Denote nonmarket time by l_{2t} , and assume that the total time endowment each period is \bar{l} .

- (a) Characterize this problem as a dynamic programming problem. What are the state variables? What are the control variables? What are the state transition equations?
- (b) Write down the Bellman equation. How do we know there is a solution to this equation?

State Variables: $\{a_t, w_t\}$. Control Variables: $\{c_{1t}, c_{2t}, l_{1t}, l_{2t}\}$. State Transition Equations: $\begin{array}{l} a_{t+1} = (1-\delta)a_t + l_{2t} \\ w_{t+1} = h(w_t) \end{array}$ Bellman Equation: $V(a, w) = \max_{c_1, c_2, l_1, l_2} \{u(c_1, c_2) + \beta V(a', w')\}$ subject to the state transition equations, the time constraint $l_1 + l_2 = \overline{l}$, and the budget constraints $c_{1t} = w_t l_{1t}$ and $c_{2t} = f(a_t)$.

5. (20 points). Search, Labor Supply, and Asset Accumulation. In the simple McCall model discussed in class, the worker could not decide how much to work, nor did he have any ability to save. This question asks you to relax those assumptions. As before, suppose each period an unemployed worker receives an offer to work forever at wage w, where w is drawn from the distribution F(w). Wage offers are identically and independently distributed over time. The worker maximizes,

$$E\sum_{t=0}^{\infty}\beta^{t}u(c_{t},l_{t}) \qquad 0<\beta<1$$

where c_t is consumption and l_t is leisure. The worker has one unit of time each period, so that $l_t + n_t = 1$, where n_t is hours worked when employed. When the worker is unemployed, $l_t = 1$, $n_t = 0$, and the worker's budget constraint is

$$a_{t+1} \le R_t(a_t + z - c_t)$$

where a_t is the workers beginning of period assets, and z is unemployment compensation. The rate of return, R_t is known at the beginning of the period, but evolves stochastically over time. Assume it is drawn from the i.i.d distribution H(R). When employed, the worker's budget constraint is

$$a_{t+1} \le R_t(a_t + w_t n_t - c_t)$$

Finally, suppose the worker can save, but cannot borrow, so that a_t is constrained to be nonnegative.

- (a) Characterize the worker's optimization problem as a dynamic programming problem. What are the state variables? What are the control variables? What are the state transition equations?
- (b) Write down two Bellman equations that summarize this dynamic programming problem, one contingent on being employed, and one contingent on being unemployed.
- (c) Will the worker's optimal policy be characterized by a reservation wage? If so, what will it depend on?

State Variables: $\{a_t, w_t, R_t, s\}$ (s = E if employed, s = U if unemployed).Control Variables: $\{c, l, n, a'\}$ (if employed) $\{accept, reject, c\}$ (if unemployed). State Transition Equations: Given in the problem by the budget constraints. Bellman Equations: 1.) $V(a, w, R, E) = \max_{c,l,n,a'} \{u(c,l) + \beta \int V(a', w, R', E) dH(R')\}$ (if employed)

subject to the given budget constraint and the time constraint, l + n = 1.

2.) $V(a, w, R, U) = \max_{acc, rej} \left\{ V(a, w, R, E), \max_c [u(c, 1) + \beta \int \int V(a', w', R', U) dH(R') dF(w')] \right\}$ where the inner max is subject to the constraint a' = R(a + z - c).

The optimal policy is to set a reservation wage, $\bar{w}(a, R)$, that depends on current assets and the current interest rate.

6. (30 points). Consider the following one-period economy. All workers have utility, u(c), where c is consumption. Each worker starts the period with assets, A. There are no private insurance markets, and workers must apply to jobs in order to find employment. A large number of competitive firms have access to a common technology, and decide: (i) whether to open a vacancy, (ii) what wage, w, to offer, and (iii) what level of specialization, $0 < \alpha < 1$, to choose for their job. The cost of posting a vacancy is Φ (the same for all jobs), and a job with specialization α produces output $g(\alpha)$, where g is an increasing, concave function.

Workers observe all wage offers and specialization decisions, and decide which job to apply to. If they get a job offer, they receive the posted wage. Otherwise they obtain unemployment compensation, z. Assume that if a job with specialization level α receives q applicants, each applicant has a probability of $(1 - \alpha)\mu(q)$ of getting the job, where $\mu(q)$ is a decreasing function, while each firm of type α has a probability of filling the vacancy of $(1 - \alpha)\eta(q)$, where η is an increasing function. (Note, when a firm chooses a job with specialization α , there is a probability of $1 - \alpha$ that any given worker will be suitable for the job, where ex ante this suitability is unknown to both the worker and the firm).

(a) Define an equilibrium for this economy. Write down a constrained maximization problem that characterizes this equilibrium. (Hint: Think 'maximize expected utility subject to zero expected profits').

An equilibrium is a collection of wage-specialization pairs in each submarket, and a set of job applicants to each submarket, $q(w, \alpha)$, such that: (1) worker's expected utility is maximized subject to a zero expected profit constraint in each submarket, and (2) expected utility is the same in all submarkets. The constrained optimization problem for a given submarket is:

$$\max_{w,\alpha,q} \{ (1-\alpha)\mu(q)u(A+w) + [1-(1-\alpha)\mu(q)]u(A+z) \}$$

subject to

$$(1-\alpha)\eta(q)[g(\alpha)-w] = \Phi$$

(Note, for simplicity, I ignore corner constraints, which may make some submarkets nonoperative).

(b) Illustrate the equilibrium for this economy in (w, α) -space using indifference curves and iso-profit schedules.

Consider a graph with α on the horizontal axis, and w on the vertical axis. Under appropriate conditions on preferences and technology, the iso-profit line will be an upward sloping concave function, and the indifference curve will be an upward sloping convex function. Equilibrium can be visualized as a point of mutual tangency.

(c) Now suppose there are N groups of workers, each with different asset levels. Describe informally the equilibrium in this case. Briefly explain how it depends on whether (absolute) risk aversion is increasing or decreasing. (Note: You do *not* need to do any math).

Now there will be a collection of indifference curves, each tangent at a different point to the same isoprofit curve. If risk aversion is decreasing in wealth (the usual case), then wealthier individuals will apply to more specialized/higher-wage jobs.