## SIMON FRASER UNIVERSITY

Department of Economics

## MIDTERM EXAM - October 31

Answer the following questions True, False, or Uncertain. Briefly explain your answers. No credit without explanation. (10 points each).

1. Search frictions cause unemployment to be too high.

UNCERTAIN. In a trivial sense, this is of course true. If people didn't have to look for jobs, unemployment would no doubt be lower. However, a more relevant and interesting approach to this question takes the costs of search as given, and asks whether search externalities cause unemployment to be too high or too low. It's not obvious, since there are externalities on both sides of the market. We showed in class that it depends on the relationship between the workers' bargaining power and the elasticity of the matching function with respect to unemployment. If they are the same, then an efficient level of unemployment results, despite the search externalities. (Hosios Condition). If workers' bargaining power is higher than this, then unemployment will indeed be too high, since firms have little incentive to create vacancies. A final take on this question could mention the process of wage setting itself. If firms can post wages, then wages retain an allocational role and can, under certain conditions, effectively internalize search externalities. This produces a so-called 'competitive search equilibrium'.
2. The random walk process, $x_{t}=x_{t-1}+\varepsilon_{t}$, where $\varepsilon_{t}$ is mean zero i.i.d., is not stationary.

TRUE. For one thing, this process does not even possess a well defined unconditional mean. The process diverges to $\pm \infty$ with probability one. Also, its unconditional variance grows over time at rate $t$.
3. Consider the following stochastic difference equation: $x_{t}=\beta E_{t} x_{t+1}+\gamma y_{t}$, where $y_{t}$ is an exogenous stationary stochastic process. This equation has a unique stationary solution only if $|\beta|>1$.

FALSE. It has a unique stationary solution only if $|\beta|<1$. If $|\beta|>1$ then there are an infinite number of stationary solutions, indexed by the sunspot process $z_{t+1}=\frac{1}{\beta} z_{t}+\epsilon_{t+1}$, where $\epsilon_{t+1}$ is an arbitrary martingale difference process that can be completely unrelated to the innovations driving the fundamentals process, $y_{t}$.

The following questions are short answer. Be sure to explain and interpret your answer. Clarity and conciseness will be rewarded.
4. (15 points). Markov Chains and Business Cycles. Consider an economy that can be in only one of two states: 'boom' or 'bust'. Label the boom state 1 , and the bust state 2 . Let $y_{t}$ be the economy's GDP, so that the law of motion for GDP is $y_{t+1}=\left(1+g_{i}\right) y_{t}$, where $g_{i}$ is the economy's growth rate when in state- $i$.
(a) Suppose the economy grows by $3 \%$ during booms, and contracts by $5 \%$ during busts. Also, suppose that we know that the economy's long-run average growth rate is $2 \%$. Using this information, calculate the long-run average probability that the economy is in a boom.

Let $\bar{\pi}_{1}$ be the long-run (stationary) probability that the process is in state 1. Given the information in the question, it is determined by the following equation

$$
\bar{\pi}_{1}(.03)+\left(1-\bar{\pi}_{1}\right)(-.05)=.02
$$

Therefore, $\bar{\pi}_{1}=7 / 8$.
(b) Suppose we also know that conditional on being in a boom, the probability of suffering a bust is $1 / 10$. Given this information, write down the Markov Chain transition matrix governing GDP fluctuations.
Let $p_{i j}$ denote the elements of the Markov transition matrix, where $p_{i j}$ represents the probability of moving from state $i$ to state $j$. One can easily show (using the fact that $\bar{\pi}^{\prime} P=\bar{\pi}^{\prime}$ defines the stationary distribution) that

$$
\bar{\pi}_{1}=\frac{p_{21}}{p_{21}+p_{12}}
$$

The question tells you that $p_{12}=1 / 10$. Therefore, given our previous calculation of $\bar{\pi}_{1}$, we can easily deduce that $p_{21}=7 / 10$. Hence, Markov transition matrix is

$$
P=\left[\begin{array}{ll}
.9 & .1 \\
.7 & .3
\end{array}\right]
$$

5. (25 points). Search with A Finite Horizon. In the simple McCall model discussed in class, the worker lived forever. This question asks you to relax that assumption. As before, suppose each period an unemployed worker receives an offer to work at wage $w$ until he dies, where $w$ is drawn from a uniform distribution between $[0,1]$. Wage offers are identically and independently distributed over time. Assume that if the worker turns down a job offer, he cannot recall it. The worker maximizes,

$$
E \sum_{t=0}^{T} \beta^{t} y_{t} \quad 0<\beta<1
$$

where $T$ is the (known) lifetime of the worker. For simplicity, suppose the worker receives no unemployment compensation, so that $y_{t}=w_{t}$ if employed, and $y_{t}=0$ if unemployed.
(a) Write down the Bellman equation that characterizes the worker's optimization problem.

Let $\tau$ denote the numbers of periods of life remaining (including the current period). Then the worker's Bellman equation is just

$$
\begin{aligned}
V(w, \tau) & =\max _{A, R}\left\{\sum_{j=0}^{\tau-1} \beta^{j} w, \quad \beta E\left[V\left(w^{\prime}, \tau-1\right)\right]\right\} \\
& =\max _{A, R}\left\{\frac{1-\beta^{\tau}}{1-\beta} w, \quad \beta \int_{0}^{1} V\left(w^{\prime}, \tau-1\right) d w^{\prime}\right\}
\end{aligned}
$$

(b) Prove that the worker's optimal policy is characterized by an age-dependent reservation wage. Derive a quadratic equation that characterizes the reservation wage. Prove that the reservation wage declines over time. Explain why. (Hints: (1) Note that to derive the value of accepting a job you need to use the formula for the partial sum (i.e, from 1 to $\tau$ ) of a geometric series, where $\tau$ is the remaining lifetime of the worker. (2) What must be the reservation wage during the final period of life? Set $\tau=1$ in your answer and check whether it gives you this common sense result. As another check on your answer, verify that $\bar{w} \rightarrow 1$ as $\beta \rightarrow 1$ and $\tau \rightarrow \infty$.
The logic of the proof that the worker's optimal policy is characterized by a reservation wage is the same as in the textbook. The two key observations are that the Accept option is linearly increasing
in $w$, while the Reject option is independent of the current $w$. For low $w$ the max will be to Reject, while for a sufficiently high $w$ the max will be to Accept. The only difference here is that the graph shifts each period, as $\tau$ changes. As a result, the worker's reservation wage will depend on the length of his remaining lifetime. We can actually calculate it by examing the indifference condition. Let $\bar{w}$ denote the reservation wage. Then, by definition, we have

$$
\frac{1-\beta^{\tau}}{1-\beta} \bar{w}=\beta\left[\int_{0}^{\bar{w}} \frac{1-\beta^{\tau-1}}{1-\beta} \bar{w} d w^{\prime}+\int_{\bar{w}}^{1} \frac{1-\beta^{\tau-1}}{1-\beta} w^{\prime} d w^{\prime}\right]
$$

Simplifying, we get

$$
\left(1-\beta^{\tau}\right) \bar{w}=\left(\beta-\beta^{\tau}\right) \bar{w}^{2}+\frac{1}{2}\left(\beta-\beta^{\tau}\right)\left(1-\bar{w}^{2}\right)
$$

Setting $\tau=1$ we can immediately see that we get the common sense result that $\bar{w}=0$ during your last period of life (remember, there is no unemployment compensation). On your last day on earth you take any job offer you get! Also, notice that when $\tau \rightarrow \infty$ the above quadratic simplifies to $\bar{w}^{2}-2 \beta^{-1} \bar{w}+1=0$ which has the solution $\beta^{-1} \pm \sqrt{\beta^{-2}-1}$. Therefore, as $\beta \rightarrow 1$ we get $\bar{w} \rightarrow 1$. This makes sense. If you're immortal, and don't discount the future, you're going to hold out for the highest wage possible. Finally, you can explicitly calculate the comparative static result, $\frac{d \bar{w}}{d \tau}$, but it is a bit ugly. The key thing you need to remember from your college calculus course is that $\frac{d}{d \tau}\left(\beta^{\tau}\right)=\beta^{\tau} \ln \beta$, which is negative, since $\ln \beta<0$ (to prove this, just take logs, and then differentiate). Note that the above quadratic can be written as follows

$$
\bar{w}^{2}-2 f(\tau) \bar{w}+1=0 \quad \text { where } \quad f(\tau)=\frac{1-\beta^{\tau}}{\beta-\beta^{\tau}}
$$

Applying the implicit function theorem, we can regard this as defining the function, $\bar{w}(\tau)$, and differentiate to obtain

$$
2 \bar{w} \frac{d \bar{w}}{d \tau}-2 f(\tau) \frac{d \bar{w}}{d \tau}-2 \bar{w} f^{\prime}(\tau)=0
$$

Using the above result to evaluate $f^{\prime}(\tau)$ and simplifying we get

$$
\left[\left(\beta-\beta^{\tau}\right) \bar{w}-\left(1-\beta^{\tau}\right)\right] \frac{d \bar{w}}{d \tau}=\bar{w} \frac{\beta^{\tau} \ln (\beta)(1-\beta)}{\beta-\beta^{\tau}}
$$

The right-hand side is negative because $\ln (\beta)<0$. The term multiplying $\frac{d \bar{w}}{d \tau}$ on the left-hand side is also negative, since $\bar{w}<1$ (making sure $\bar{w}<1$ forces us to choose the smaller of the two roots of the quadratic). Hence, in a long-winded manner, we have established the common sense result, $\frac{d \bar{w}}{d \tau}>0$. That is, the longer your horizon, the more selective you are in accepting a job. (Of course, in the real world, young people are often less selective than older workers, but that obviously reflects factors that are missing from this model).
6. (30 points). Deposits on Workers. Recently there have been many proposals to lower the currently high unemployment rates in many countries. One of them involves having firms pay a 'deposit' every time they lay off a worker, which will be refunded when a new worker is hired. Let's use the MortensenPissarides model the analyze the consequences of this plan. Specifically, start with the baseline MP model. Time is discrete, and $\beta=1 /(1+r)$ is the rate of interest and time preference. Matches between workers and firms take place via the 'matching function' $M(u, v)=u^{\alpha} v^{1-\alpha}$. Each worker produces $y$ units of output while employed, and enjoys $z$ units of leisure while unemployed. Firms must incur a vacancy cost of $c$ each period they search for a worker. Each period there is a constant probability, $s$, of a match being destroyed. As usual, suppose that wages are set via Nash Bagaining, and let $\phi$ denote the relative bargaining power of workers, and let $w$ denote the wage. Also as usual, suppose there is free entry and exit so that the equilibrium value of creating a vacancy is zero.
(a) First suppose there is no worker deposit. Let $J$ denote the value of an operating firm and $V$ denote the value of a searching firm. Write down 2 Bellman equations that describe the firm's problem. Let $E$ denote the value function of an employed worker, and $U$ denote the value function of an unemployed worker. Write down 2 Bellman equations that describe the worker's problem.
This part of the question is straight from the textbook. The firm's Bellman equations are

$$
\begin{aligned}
J & =y-w+\beta[s V+(1-s) J] \\
V & =-c+\beta\{q(\theta) J+[1-q(\theta)] V\}
\end{aligned}
$$

and the worker's Bellman equations are

$$
\begin{aligned}
E & =w+\beta[s U+(1-s) E] \\
U & =z+\beta\{\theta q(\theta) E+[1-\theta q(\theta)] U\}
\end{aligned}
$$

(b) Show that a steady state equilibrium for this economy can be described by the following 3 equations:

$$
\begin{aligned}
u & =\frac{s}{s+\theta q(\theta)} \\
w & =y-\frac{(r+s) c}{q(\theta)} \\
w & =z+\phi(y-z+\theta c)
\end{aligned}
$$

where $\theta=v / u$ denotes labor market 'tightness'.
Again, this is from the textbook (see pgs. 948-949). I won't repeat the details. The idea is to use the firm's Bellman equations, along with the free entry condition, $V=0$, to derive the Zero Profit/Job Creation curve (given by the middle equation above). Then use the Nash Bargaining result, $E-U=\phi S$ and $J=(1-\phi) S$ (where $S=E-U+J)$, to derive the the Wage Curve given by the bottom equation.
(c) Now let's consider the deposit plan. Suppose each time a match is destroyed the firm must pay a lump-sum cost of $\delta$. However, suppose that each time a match is created the firm receives a lump-sum payment of $\delta$. Re-write the firm's Bellman equations. Re-derive the Wage Curve and the Job Creation Curve. Use a graph to illustrate how the deposit plan will affect $\theta$, $w$, and $u$. According to the MP model, will this plan work? Explain.
With the deposit plan, the firm's Bellman equations become

$$
\begin{aligned}
J & =y-w+\beta[s(V-\delta)+(1-s) J] \\
V & =-c+\beta\{q(\theta)(J+\delta)+[1-q(\theta)] V\}
\end{aligned}
$$

(Note that the worker's Bellman equations remain the same). From the free entry condition, $V=0$, and the second Bellman equation, we now obtain

$$
J=\frac{c}{\beta q(\theta)}-\delta
$$

Note that free entry dissipates the value of the hiring subsidy. Substituting this result into the first Bellman equation gives

$$
\frac{c}{\beta q}-\delta=y-w+\beta\left[-s \delta+(1-s)\left[\frac{c}{\beta q}-\delta\right]\right]
$$

Collecting terms gives us the revised Job Creation curve

$$
w=y+(1-\beta) \delta-\frac{(r+s) c}{q}
$$

Note that an increase in $\delta$ shifts up the curve. Let's now turn to the Wage Curve. Nash Bargaining now delivers the results, $E-U=\phi S$ and $(J+\delta)=(1-\phi) S$. These imply

$$
E-U=\frac{\phi}{1-\phi}(J+\delta)
$$

The worker's Bellman equation for $E$ implies $E=\frac{1+r}{r+s} w+\frac{s}{r+s} U$, while the firm's Bellman equation for $J$ implies $J=\frac{1+r}{r+s}(y-w)-\frac{s}{r+s} \delta$. Substituting these in and simplifying yields

$$
w-\frac{r}{1+r} U=\frac{\phi}{1-\phi}\left[(y-w)+\frac{r}{1+r} \delta\right]
$$

To eliminate $\frac{r}{1+r} U$, note that the worker's Bellman equation for $U$ implies $E-U=\frac{r U-(1+r) z}{\theta q}$. Combining this with the Nash Bargaining solution $E-U=\frac{\phi}{1-\phi}(J+\delta)$, along with the previous result for $J$ gives us $\frac{r}{1+r} U=\frac{\phi}{1-\phi} c \theta+z$. Substituting this in yields the revised Wage Curve

$$
\begin{aligned}
w & =z+\phi\left[y-z+c \theta+\frac{r}{1+r} \delta\right] \\
& =z+\phi[y-z+c \theta+(1-\beta) \delta]
\end{aligned}
$$

As with the Job Creation curve, an increase in $\delta$ shifts the curve up. But notice that now the upward shift is less, since it is multiplied by $\phi$. Hence, just like the case of an increase in productivity, the downward-sloping JC curve shifts up by more than the upward-sloping WC curve. Both wages and labor market tightness, $\theta$, increase. Translating the increase in $\theta$ over to the Beveridge Curve, we then find that equilibrium unemployment declines. The plan works! Job creation is stimulated since workers only obtain part of the subsidy. (Before getting too excited, note that his plan abstracts from many real world complications).
(d) What are the long-run fiscal consequences of this plan? Will the government need to raise taxes to pay for it?
In equilibrium, due to flow balance, this plan should pay for itself. That is, the number of workers hired should match the number laid off. However, there will be one-time start-up cost. Since the whole point of the plan is to drive down unemployment, during the transition there will be more workers hired than fired. The government will need to finance the net deposit payments to firms. Presumably, the resulting reduction in UI payments could help along this dimension.

