## PROBLEM SET 1 - Due October 14

1. The CAPM predicts that everyone holds the same portfolio of risky assets (ie, the market portfolio). Why is this? Why wouldn't more risk averse individuals hold a greater proportion of safe stocks?
2. Suppose a stock has a $\beta$ of 1.5 , the market return is $10 \%$, and the risk-free rate is $5 \%$. According to the CAPM, what is the expected rate of return on this stock? If the expected price a year from now is $\$ 100$, what should the price be today?
3. Assume the expected rate of return on the market portfolio is $23 \%$ and the risk-free rate (eg, the rate of return on government T-Bills) is 7\%. The standard deviation of the market portfolio is $32 \%$. Assuming the CAPM holds (ie, the market portfolio is efficient),
(a) What is the equation of the "capital market line"?
(b) If you desire a $39 \%$ expected return, what will be the associated standard deviation of this position? If you have $\$ 1,000$ to invest, how should you allocate it to achieve this position?
(c) If you invest $\$ 300$ in T-Bills and $\$ 700$ in the market portfolio, how much money can you expect to have at the end of the year?
4. On the planet Vulcan there are only two risky stocks, A and B, with the following characteristics:

|  | Number of Shares <br> Outstanding | Price <br> per share | Expected <br> rate of return | St. Dev. of <br> of return |
| :--- | :---: | :---: | :---: | :---: |
| Stock A | 100 | $\$ 1.50$ | $15 \%$ | $15 \%$ |
| Stock B | 150 | $\$ 2.00$ | $12 \%$ | $9 \%$ |

The correlation coefficient between the returns of the two stocks is $\rho=\frac{1}{3}$. There is also a risk-free asset on Vulcan, and amazingly, the CAPM holds exactly.
(a) What is the expected rate of return on the market portfolio?
(b) What is the standard deviation of the return on the market portfolio?
(c) What is the beta of Stock A?
(d) What is the risk-free rate on Vulcan?
5. True or False: If Bill and Ted have the same income and preferences, and there is a risky project which neither of them is willing to do on his own, they will not be willing to do it on a 50-50 basis either, since splitting the project halves both the return and riskiness of the project.
6. There are assets with highly volatile returns but with average returns equal to the risk-free rate. How can this be? Why would anyone hold such an asset? Wouldn't it be better to put your money in a mean-variance frontier asset?
7. True or False: It is less risky to invest half your money in the stock market every year than it is to invest all your money every other year.
8. Consider a world with just two 'states' - Rain or Shine. There are two firms in the economy - an Umbrella manufacturer and a Sunglasses manufacturer. The share price of the Umbrella manufacturer is $\$ 7$ and the share price of the Sunglasses manufacturer is $\$ 4$. Assume their state-contingent profits are as follows:

|  | Rain | Shine |
| :--- | :---: | :---: |
| Umbrellas | 2 | $1 / 2$ |
| Sunglasses | 0 | 2 |

(a) What are the (implicit) state-contingent claims prices (ie, the price of $\$ 1$ if and only if a given state occurs)?
(b) Given your answer to part (a), what must be the price of a risk-free asset?
9. Assume a representative agent maximizes a constant relative risk aversion utility function:

$$
E_{t} \sum_{j=0}^{\infty} \beta^{j} c_{t+j}^{1-\gamma}
$$

Also assume that there is a single asset in the economy that produces an exogenous stream of nonstorable dividends, $d_{t+j}$. Assume this is the only good in the economy, so that in equilibrium, $c_{t}=d_{t}$, for all $t$.
(a) Show that with $\log$ utility (ie, $\gamma=1$ ), the equilibrium price/dividend ratio is constant, no matter what the distribution of dividends is. What is the economic explanation for this result? (Hint: What are the income and substitution effects? How do they depend on $\gamma$ ?)
(b) It is often observed that stock prices go down following favorable news about future dividends or profits. The usual explanation attributes this to fears about future tightening of monetary policy in response to the news. Use this model to provide an alternative explanation. That is, under what conditions would higher future dividends lead to lower current stock prices? (Hint: think in terms of income and substitution effects. If you can't do the math, then just explain intuitively).

