SIMON FRASER UNIVERSITY
Department of Economics

Econ 809 | Prof. Kasa
Advanced Macroeconomic Theory | Spring 2005

PROBLEM SET 1 - Time Series, Markov Chains, and Dynamic Programming
(Due February 8)

1. Use the MATLAB program **bigshow.m** to simulate the following time series processes and to report their impulse response functions and their (log) spectral densities.

   (a) $x_t = \varepsilon_t$

   (b) $x_t = \varepsilon_t + .5\varepsilon_{t-1}$

   (c) $x_t = .9x_{t-1} + \varepsilon_t$

   (d) $x_t = 1.2x_{t-1} - .3x_{t-2} + \varepsilon_t$

   (e) $x_t = 1.2x_{t-1} + .3x_{t-2} + \varepsilon_t$

   (f) $x_t = .8x_{t-1} - .5x_{t-2} + \varepsilon_t$

   where ε_t is an iid shock. Briefly comment on the stochastic properties of these processes. Which exhibit business cycle features? Which are stationary? Do any possess interior maxima in their spectral densities? If so, what is the frequency of the implied cycle?

2. Do Exercise 2.14 and 2.17 (parts a. through e. only) in Ljungqvist & Sargent.

3. Do Exercises 5.1 in Ljungqvist & Sargent.

4. At the beginning of each period, a worker can choose to work at her last period's wage or draw a new wage. If she draws a new wage, the old wage is lost and she must wait one period before she can start her new job. New wages are i.i.d. draws from the c.d.f. F, where $F(0) = 0$, $F(B) = 1$ for $B < \infty$. The worker seeks to maximize $E_0 \sum_{t=0}^{\infty} \beta^t w_t$, where w_t is the wage in period-t.

   (a) Write down the Bellman equation for the worker. (Hint: the value function appears in both options available to the worker).

   (b) Now suppose the worker receives an unemployment compensation of c. What is the Bellman equation now?

   (c) Assume wage offers are distributed uniformly on the interval $[0, 1]$. Use the program **ex2.m** to numerically solve the Bellman equation for alternative values of β and c. (This program first discretizes things and then uses a value function iteration method. The value function iteration is done by calling the program **valit.m**.) How does the reservation wage vary with β and c? Interpret the results.

5. This problem asks you to compute the solution to a standard stochastic growth model using two alternative numerical methods. The first is a standard linear quadratic approximation. The second is a projection method. It expands the value function in terms of Chebyshev polynomials, and computes the basis coefficients via collocation. To do this you need to access the MATLAB files in the compecon toolbox, which is based on the text *Applied Computational Economics and Finance*, by Mario Miranda and Paul Fackler. Go to the course webpage and download and unzip the file **compecon.zip**. Some of the programs use C code, and you will need to first run the program *mexall* to create the necessary MATLAB readable files. Once you've done this, just run the program **demdp07.m**. (Note: there are many interesting and useful demo programs in this package. You may want to play around with others.)

   The model is as follows. There is a representative agent who produces and consumes a single composite good. The stock of the good available at the beginning of period-t is s_t, and its law of motion is given by:

$$s_{t+1} = \gamma x_t + \epsilon_{t+1} h(x_t)$$

   where x_t is the amount invested, γ is the survival rate of capital (1 minus the depreciation rate), h is the production function, and ϵ is a productivity shock with a mean of 1. Hence, the Bellman equation is given by:

$$V(s) = \max_{0 \le x \le s} \{u(s-x) + \delta E_\epsilon V(\gamma x + \epsilon h(x))\}$$

   For this particular problem, assume the utility function takes the CRRA form $u(c) = c^{1-\alpha}/(1-\alpha)$, and that the production function takes the Cobb-Douglas form $h(x) = x^\beta$. Set $\alpha = 0.2$ and $\beta = 0.5$. Also assume the productivity shock is lognormal$(0, \sigma^2)$, and set $\sigma = 0.1$, $\gamma = 0.9$, and $\delta = 0.9$.

   As discussed in class, the projection method approximates the value function as $V(s) \approx \sum_{j=1}^{n} c_j \phi_j(s)$, where $\phi_j(s)$ are Chebyshev polynomials, and the c_j coefficients solve the collocation equation:

$$\sum_{j=1}^{n} c_j \phi_j(s_i) = \max_{0 \le x \le s_i} \left\{ u(s_i - x) + \delta \sum_{k=1}^{K} \sum_{j=1}^{n} w_k c_j \phi_j(\gamma x + \epsilon_k h(x)) \right\}$$

   where ϵ_k and w_k are quadrature nodes and weights for the discrete approximation of the lognormal shock. For this problem, use a three-node Gaussian quadrature by setting $nshocks = 3$. Also, try a 10-function Chebyshev polynomial basis (set $n = 10$) on the interval $[5, 10]$ (set $smin = 5$ and $smax = 10$).

   Briefly describe and interpret the policy and value functions. Are their any differences between the Linear-Quadratic approximation and the Chebyshev approximation?

6. Read the first 6 or 7 pages of the paper *Solution of Macromodels with Hansen-Sargent Robust Policies* by Giordani and Soderlind, which discusses recent work on the concepts of "model uncertainty" and "robust policies". Consider the following dynamic model:

$$\begin{aligned} x_{t+1} &= b_1 x_t - b_2(i_t - \pi_t) + u_{t+1} && (1) \\ \pi_{t+1} &= \pi_t + a_1 x_t + e_{t+1} && (2) \end{aligned}$$

where x_t denotes the "output gap", π_t is inflation, and i_t is the nominal interest rate. Versions of this model are often used in the applied monetary policy literature. Suppose a central banker has the following objective function:

$$\min_{\{i_t\}} E_0 \sum_{t=0}^{\infty} \beta^t [\gamma_1 x_t^2 + \gamma_2 \pi_t^2 + \gamma_3 i_t^2 + \gamma_4 (i_t - i_{t-1})^2]$$

where the γ_i parameters are weights on deviations of the various variables from their targets (normalized to be zero).

Now, suppose the central banker isn't sure that eqs. (1) and (2) are the "true model", and so he wants to formulate a policy that is "robust" to general nonparametric forms of misspecification. To implement this, run the program **Test1.m** using the default parameter values. This program should give you impulse response functions of x_t, π_t, and i_t in response to the aggregate demand shock u_t. There are two responses plotted for each variable. One shows the response of the baseline "approximating" model and the other shows the response of the "worst case" model.

Run the program for two alternative values of the θ parameter, which summarizes the degree of uncertainty and/or the preference for robustness. First try $\theta = 10$ and then try $\theta = 3$. (This parameter is set in the **Test1.m** program). Compare and interpret the results.