SIMON FRASER UNIVERSITY
Department of Economics

Econ 842 | Prof. Kasa
International Monetary Economics | Spring 2014

PROBLEM SET 1 - CURRENT ACCOUNT DYNAMICS
(Due February 20)

1. (25 points). Consider a 2-period world economy consisting of two countries. Each has preferences

$$U(C_1, C_2) = \sqrt{C_1} + \sqrt{C_2}$$

The Home country has endowments $Q_1 = 1$ and $Q_2 = 2$. The Foreign country has endowments $Q_1^* = 2$ and $Q_2^* = 1.3$. Both countries have open capital markets, and both begin with zero net foreign assets.

(a) Compute the equilibrium world interest rate. (Hint: Equilibrium requires $S(r) + S^*(r) = 0$, where $S(r)$ and $S^*(r)$ are the Home and Foreign saving functions, e.g., $S(r) = Q_1 - C_1(r)$).

(b) Given this interest rate, what are the equilibrium values of Home consumption, C_1 and C_2. Use the above utility function to then compute Home utility.

(c) Now suppose the Foreign country experiences a higher growth rate. In particular, suppose $Q_2^* = 2.5$, with all other endowments remaining the same. What is the new world interest rate? What is Home utility now? Is Foreign growth good or bad for the Home country? Explain.

2. (25 points). This question examines the implications of a simple modification of the linear-quadratic current account model discussed in class. To set the stage, consider a one-good/one-asset world consisting of a large number of identical countries, each inhabited by a representative agent with preferences

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right\} \tag{1}$$

where $u(C_s) = -(a - C_s)^2$. Residents of each country receive a stochastic endowment sequence, $\{Y_s\}$, of the single good. Endowment innovations are independent across countries, giving rise to potential risk-sharing gains. However, assume the only mechanism available for doing this is via ex-post trade in one-period riskless bonds, B_t.

Consider first the case of a single country facing given world market conditions. In particular, suppose the world interest rate is constant, and given by r. Hence, the agent's budget constraint is

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s \tag{2}$$

where B_t is the agent's initial stock of foreign assets. (Note, this constraint must hold with probability one).

(a) Assuming $\beta(1+r) = 1$, write down the agent's Euler equation for consumption.

(b) Substitute this into the agent's budget constraint and derive the agent's consumption function (in terms of the path of expected future endowments). (Note: since the budget constraint holds with probability one, it also holds in expectation).

(c) Using the consumption function in part (b), derive an expression for the current account. Interpret this expression (i.e., explain under what circumstances the country will run current account surpluses or current account deficits).

Now let's modify the above model by assuming that agents have preferences of the following recursive form:

$$U_t = u(C_s) + \beta \frac{2}{\sigma} \log E_t[\exp(\frac{\sigma}{2} U_{t+1})] \tag{3}$$

where $\sigma \leq 0$ is a parameter related to risk aversion, and $u(C_s)$ continues to be the quadratic expression $u(C_s) = -(a - C_s)^2$. (Note: this is an example of a class of non-state separable preferences popularized by Epstein and Zin (*Econometrica*, 1989), in their work on asset pricing). For analytical convenience, suppose that endowments follow the process, $Y_t = \bar{Y} + \varepsilon_t$, where $\varepsilon_t \sim i.i.d.\ N(0,1)$. Also, define the change of variable $\tilde{C}_s = a - C_s$, and the adjusted wealth variable

$$W_t = R \cdot B_t + Y_t + \frac{\bar{Y} - Ra}{r} \tag{4}$$

where $R = (1+r)$. Using these definitions, we can restate the agent's problem as

$$U_t = \arg\max_{\tilde{C}} \left\{ -\tilde{C}_t^2 + \beta \frac{2}{\sigma} \log E_t \left[\exp\left(\frac{\sigma}{2} U_{t+1} \right) \right] \right\} \tag{5}$$

subject to $W_{t+1} = R(W_t + \tilde{C}_t) + \varepsilon_{t+1}$. Conjecture that $U_{t+1} = -PW_{t+1} - \kappa$, where P and κ are unknown (positive) constants to be determined (via standard fixed point procedures). Then it can be verified that

$$\tilde{C}_t = -FW_t \qquad \text{where} \qquad F = \frac{R \cdot D(P)}{1 + R \cdot D(P)} \tag{6}$$

and $D(P) = P/(1+\sigma P)$. Solving the Bellman/Riccati fixed point equation yields $P = r/(\sigma + R)$. (Note: you do not need to derive these results, unless of course you like doing algebra). Finally, using these results, we get the following laws of motion for (adjusted) wealth and the current account

$$W_t = R(1-F)W_{t-1} + \varepsilon_t \tag{7}$$

$$CA_t = \left(\frac{r}{R} - F \right) W_t + \frac{1}{R} \left(Y_t - \bar{Y} \right) \tag{8}$$

Note that in the standard Linear-Quadratic/Certainty-Equivalence case, where $\sigma = 0$, we get the usual result that consumption and wealth follow random walks, and the current account is independent of W_t.

(d) Briefly describe how adjusted wealth and the current account behave when $\sigma < 0$. Explain the intuition. Relate your discussion to the literature on precautionary saving.

Now let's take advantage of an alternative interpretation of the above preferences, using results from Hansen and Sargent's recent monograph entitled *Robustness*. They show that

$$\frac{2}{\sigma}\log E_t\left[\exp\left(-\frac{\sigma}{2}PW_{t+1}\right)\right] = -R^2(W_t+\tilde{C}_t)^2\left(\frac{\theta P}{\theta - P}\right) = \arg\min_{z_{t+1}}[\theta z_{t+1}^2 - R^2(W_t+\tilde{C}_t+z_{t+1})^2 P]$$

where $\theta = -\sigma^{-1}$. Using this, we can recast the agent's problem as a (deterministic) dynamic zero-sum game, with the associated Bellman-Isaacs equation

$$-W_t^2 P = \max_{\tilde{C}_t}\min_{z_{t+1}}[-\tilde{C}_t^2 + \beta\theta z_{t+1}^2 - \beta W_{t+1}^2 P] \tag{9}$$

subject to $W_{t+1} = R(W_t + \tilde{C}_t) + z_{t+1}$. The idea here is that the agent is uncertain (in the Knightian sense) about the stochastic endowment process, so he wants to devise a 'robust' decision rule. As a mechanism for doing this, he imagines a malicious agent chooses a disturbance process, z_{t+1}, so as to subvert his control efforts. That is, the agent plays a game against himself. The parameter θ determines how much freedom the 'evil agent' has. As $\theta \uparrow \infty$, the evil agent's actions become increasingly costly, and so the solution converges to the standard one. Hence, lowering θ produces a more robust decision rule. (Hansen and Sargent describe procedures for calibrating this parameter).

(e) Describe how this economy would respond to a sudden increase ambiguity or uncertainty. In particular, suppose that initially $W_0 < 0$ (below the long-run steady state), and then suddenly θ decreases. How do wealth and the current account respond? How might this result apply to the aftermath of the Asian crisis? (Hint: Refer to recent developments in real interest rates and global current account imbalances).

(f) Compare and contrast this account of recent global imbalances to the work of Caballero, Farhi, and Gourinchas (*AER* 2008) posted on the course webpage.

3. (20 points). Using the data on the webpage, and whatever software you want, report plots of the current account, as a fraction of GDP, for the U.S., U.K, Japan, and Canada.

4. (30 points). Pick a country, and following the procedure outlined on pages 90-93 of the Obstfeld-Rogoff text, test the Present-Value Model of the current account (i.e., test the model's implied cross-equation restrictions). Plot the model's predicted current account against the actual current account. Comment on the model's fit. (Note: Be sure to express everything in real terms. Although variables should also be expressed in per capita terms as well, don't worry about that. It shouldn't make much of a difference here).