SIMON FRASER UNIVERSITY
Department of Economics

Econ 808 | Prof. Kasa
Macroeconomic Theory | Fall 2010

PROBLEM SET 2 - Search and Matching
(Due October 22)

1. An unemployed worker samples wage offers on the following terms: Each period, with probability ϕ, $0 < \phi < 1$, she receives no offer (you can regard this as a wage offer of zero forever). With probability $1 - \phi$ she receives an offer to work for w forever, where w is drawn from the cdf $F(w)$. Successive draws are independently and identically distributed over time. The worker chooses a strategy to maximize

$$E \sum_{t=0}^{\infty} \beta^t y_t$$

where $y_t = w$ if the worker is employed, and $y_t = c$ if the worker is unemployed. Assume that if a job offer is accepted, the worker stays on the job forever.

Let $v(w)$ be the expected value of $\sum_{t=0}^{\infty} \beta^t y_t$, for an unemployed worker who has offer w in hand and who behaves optimally from now on. Write down the Bellman equation for the worker's problem.

2. Each period an unemployed worker receives an offer to work forever at wage w_t, where $w_t = w$ in the first period and $w_t = \phi^t w$ after t periods on the job. Assume wages increase with tenure, ie, $\phi > 1$. Also assume the initial wage offer is drawn from a distribution $F(w)$ that is constant over time (ie, entry-level wages are stationary).

The worker's objective is to maximize $E \sum_{t=0}^{\infty} \beta^t y_t$ where $y_t = w_t$ if the worker is employed and $y_t = c$ if the worker is unemployed, where c can be interpreted as unemployment compensation. Let $v(w)$ be the optimal value of the objective function for an unemployed worker who has offer w in hand. Write down the Bellman equation for this problem. Show that, if two economies differ only in the growth rate of wages, eg, $\phi_1 > \phi_2$, then the economy with the higher wage growth has a lower reservation wage. Interpret the result. (Note: assume $\beta\phi_i < 1$, $i = 1, 2$).

3. Each period an unemployed worker receives an offer to work forever at wage w, where w is drawn from the distribution $F(w)$. Offers are i.i.d. Each worker also has another source of income, denoted by ϵ_t, which can be interpreted as financial/nonhuman wealth. Each period workers get a realization of ϵ_t, which is i.i.d., and is drawn from the distribution $G(\epsilon)$. Also assume that w_t and ϵ_t are independently distributed. A worker's objective is to maximize

$$E \sum_{t=0}^{\infty} \beta^t y_t$$

where $y_t = w + \phi\epsilon_t$ if the worker has accepted a job with wage w, and $y_t = c + \epsilon_t$ if the worker remains unemployed. To reflect the fact that an employed worker has less time to collect information on nonhuman wealth, assume that $\phi < 1$. Also assume $0 < \text{prob}[w \geq c + (1-\phi)\epsilon] < 1$.

Write down the worker's Bellman equation, and prove that the reservation wage increases with the level of nonhuman wealth. Interpret the result.

4. Consider the benchmark Mortensen-Pissarides model discussed in class. Steady state equilibrium in this model takes the form of 3 equations (e.g., eqs. 26.3.1, 26.3.5, and 26.3.12 in Ljungqvist & Sargent) in the 3 endogenous variables u, w, and θ. These equations are sometimes referred to as the Beveridge Curve, the Job Creation Curve, and the Nash Bargaining Curve, respectively. Using this system, compute the comparative static effects of (permanent) changes in productivity, y, and the job separation rate, s. Interpret these results economically. [Hints: (1) Notice that you can first consider separately the 2-dimensional system (26.3.5 and 26.3.12) for w and θ, (2) Check your qualitative results graphically using 2 graphs, one in (θ, w)-space and one in (v, u)-space.

5. Consider the following 1-period economy. There are two types of workers. Fraction λ are risk averse, with concave utility function $u(c)$. Fraction $1-\lambda$ are risk-neutral, with linear utility function $v(c) = c$. Both types of workers produce output $f(k)$ when matched with a firm employing capital k, where f satisfies the usual assumptions. There is a large number of potential firms which can enter, buy capital (at price 1 per unit), and post vacancies.

   Matching takes place as follows: First, firms decide whether to enter, irreversibly buy some capital, and post wages. Then, workers observe all advertised wages, and decide which jobs to apply for. If employed, a worker receives the promised wage (i.e., ex post renegotiation is not permitted), otherwise he receives some unemployment benefit, z. Workers are assumed to make their application decisions without coordination, which leads to the usual matching frictions. In particular, if qN workers apply to N firms (i.e., N firms offer some wage w', and qN workers seek wage w'), then the firm gets a worker with probability $1 - e^{-q}$, and each worker is employed with probability $(1 - e^{-q})/q$. (In case you're interested, this is the limit of a standard urn-ball process as $N \to \infty$). Note that workers have to trade-off wages and employment probabilities when formulating their application decisions. Also, note that because firms must choose their capital before matching, if a firm does not get a worker, its capital is sunk.

   (a) Define an equilibrium. Show that the equilibrium can be characterized by a pair of constrained maximization problems. (Hint: Think "max expected utility subject to zero expected profits").

   (b) Characterize the equilibrium, and show that (generically) there will be an observed wage distribution with just two wages, w^h and w^l (where $w^h > w^l$), with equilibrium fractions $1-\mu$ and μ, respectively.

   (c) Illustrate this equilibrium graphically. (Hint 1: Draw the workers' indifference curves in (q, w) space. Plug the firms' first-order condition for k into the zero expected profit condition to get a zero-profit locus of (q, w) combinations. Now combine the indifference curves with the zero profit locus. Hint 2: Remember Moen (JPE, 1997)!).