

SIMON FRASER UNIVERSITY
Department of Economics

Econ 809
Advanced Macroeconomic Theory

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PROBLEM SET 3 - Complete Markets, Overlapping Generations, and Asset Pricing
(Due March 29)

1. Consider a complete markets economy with a representative agent. There is one good in the economy, which arrives as an exogenous endowment following the process:

$$y_{t+1} = \lambda_{t+1}y_t$$

where y_t is the endowment at time- t and $\{\lambda_{t+1}\}$ follows a two-state Markov chain with transition matrix

$$P = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix}$$

and initial distribution $\pi_\lambda = [\pi_0 \quad 1 - \pi_0]$. The value of λ_t is given by $\bar{\lambda}_1 = 0.98$ in state 1 (the ‘bad’ state) and $\bar{\lambda}_2 = 1.03$ in state 2 (the ‘good’ state). Assume the history of y_s, λ_s up to t is observed at time- t . The consumer ranks consumption sequences according to the utility function $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$ where $\beta \in (0, 1)$ and $u(c) = c^{1-\gamma}/(1-\gamma)$, where $\gamma \geq 1$.

- (a) Carefully define a competitive equilibrium, and describe how to compute it.

In what follows, assume $p_{11} = .8$, $p_{22} = .85$, $\pi_0 = .5$, $\beta = .96$, and $\gamma = 2$. Suppose the economy begins with $\lambda_0 = .98$ and $y_0 = 1$.

- (b) Compute the unconditional average growth rate of consumption (i.e., before observing λ_0).
- (c) Compute the time-0 prices of three risk-free discount bonds, i.e., those promising to pay one unit of time- j consumption for $j = 0, 1, 2$, respectively.
- (d) Compute the time-0 prices of three state-contingent bonds, i.e., those promising to pay one unit of time- j consumption contingent on $\lambda_j = \bar{\lambda}_1$ for $j = 0, 1, 2$.
- (e) Compute the time-0 prices of three state-contingent bonds, only this time assume they are contingent on state 2, again at dates $j = 0, 1, 2$.
- (f) Compare the results from parts c, d, and e. Interpret the results.

2. Consider the following overlapping generations model of two-period-lived agents. At each date $t \geq 1$ N_1 type-1 individuals are born, each endowed with $y > 0$ units of the consumption good when young and zero units old. At the same time, N_2 type-2 agents are born who are endowed with zero units of the consumption good when young and $Y > 0$ units when old. The consumption good is nonstorable. At time $t = 1$, there are

N old people, all of the same type, each endowed with zero units of the consumption good and H_0/N units of unbacked paper, called “fiat currency”. The populations of type-1 and type-2 agents remain constant over time. Preferences are identical, and given by $U(c_t^h(t), c_t^h(t+1)) = \ln c_t^h(t) + \ln c_t^h(t+1)$, where $c_t^h(s)$ is consumption at date s of an agent h born at time- t .

- (a) Consider an equilibrium without valued currency (i.e., an equilibrium without intergenerational trade). Let $[1 + r(t)]$ be the gross rate of return on consumption loans. Find a formula for $[1 + r(t)]$ as a function of N_1, N_2, y , and Y .
 - (b) Suppose that N_1, N_2, y , and Y are such that $[1 + r(t)] > 1$ in the equilibrium without valued currency. Prove that there does not exist quantity-theoretic equilibrium where fiat currency is valued and where the price level $p(t)$ obeys the quantity theory equation $p(t) = q \cdot H_0$, where q is proportionality constant and $p(t)$ is measured in units of currency per good.
 - (c) Now suppose that N_1, N_2, y , and Y are such that in the nonmonetary equilibrium $[1 + r(t)] < 1$. Prove that there exists an equilibrium in which fiat currency is valued and the price level obeys the quantity equation $p(t) = q \cdot H_0$ for some constant q . Outline an argument showing that the equilibrium with valued currency is not Pareto superior to the nonmonetary equilibrium.
 - (d) Suppose that N_1, N_2, y , and Y are such that, in the preceding nonmonetary equilibrium, (i.e., where $[1 + r(t)] < 1$), there exists an equilibrium in which fiat currency is valued. Now consider an alternative economy, identical to the preceding one in all respects except for the following feature: each period a government purchases a constant amount, L_g , of private consumption loans and pays for them by issuing debt on itself, called “inside money” M_I , in the amount $M_I(t) = L_g \cdot p(t)$. The government never retires this inside money, instead using the proceeds of the loans to finance new purchases of consumption loans in subsequent periods. The quantity of “outside money”, or currency, remains H_0 , so that the total money stock is now $H_0 + M_I$.
 - (i) Show that in this economy there exists a monetary equilibrium in which the price level is constant over time, with $p(t) = q \cdot H_0$, where q is defined in part (c) above.
 - (ii) Briefly explain why government purchases of private debt (i.e., ‘open-market operations’) are not inflationary in this economy.
 - (iii) In standard macro models, one-time open market operations in private debt normally affect real variables and/or the price level. What accounts for the differences between those models and this one?
3. Consider an economy (“economy A”) consisting of two-period-lived overlapping generations. At each date $t \geq 1$ a constant number N of young people are born, who want to consume both when they are young, at t , and when they are old, at $(t + 1)$. Each young person has the utility function $\ln c_t(t) + \ln c_t(t + 1)$, where $c_s(t)$ is the time- t consumption of an agent born at s . For all dates $t \geq 1$, young people are endowed with $y > 0$ units of a single nonstorable good when they are young and zero units when they are old. In addition, at time $t = 1$ there is an initial old generation comprised of

N people, each endowed with H/N units of unbacked fiat currency. Let $p(t)$ denote the nominal price level at t , denominated in dollars per time- t goods.

- (a) Define and compute an equilibrium with valued fiat currency. Argue that it is unique.

Now consider a second economy (“economy B”) that is identical to economy A except that in economy B there is a pay-as-you-go social security system. In particular, at each date $t \geq 1$, the government taxes $\tau > 0$ units of the time- t consumption good away from each young person and at the same time gives τ units of the time- t good to each existing old person.

- (b) Does economy B possess an equilibrium with valued fiat currency? Describe the restrictions on τ , if any, that are needed to guarantee the existence of such an equilibrium.
- (c) If a monetary equilibrium exists, is it unique?
- (d) Now consider *stationary* monetary equilibria. Is there only one, or are there multiple stationary monetary equilibria? Describe how the price level varies across economies with different social security systems, as measured by τ .

4. Consider an economy consisting of two-period-lived overlapping generations. At each date $t \geq 1$, N_1 “lenders” are born who are endowed with $\alpha > 0$ units of the single nonstorable consumption good when they are young and zero units when they are old. There are also born N_2 “borrowers” who are endowed with zero units of the consumption good when young and $\beta > 0$ when old. Assume N_1 and N_2 are constant and satisfy the restriction $N_2\beta < N_1\alpha$. Assume everyone has preferences $\ln c_t(t) + \ln c_t(t+1)$, where $c_t(s)$ is consumption at time- s of an agent born at time- t . Finally, as usual, there is an initial old generation at time $t = 1$, consisting of N agents, each endowed with $H(0)/N$ units of unbacked fiat currency.

- (a) Compute the market-clearing interest rate on consumption loans in the nonmonetary equilibrium.
- (b) Outline a brief argument to establish whether or not the nonmonetary equilibrium is Pareto optimal.

Now assume there is a government that purchases (and destroys) G_t units of the good in period- t . The government finances its purchases entirely by money creation. That is, at each time- t ,

$$G_t = \frac{H(t) - H(t-1)}{p(t)}$$

where $H(t) - H(t-1)$ is the additional currency printed by the government in period- t and $p(t)$ is the price level at t . The government increases $H(t)$ according to

$$H(t) = zH(t-1) \quad z \geq 1$$

where z is constant. At time- t the old generation offers $H(t-1)$ units of currency in exchange for time- t goods, while at the same time the government offers $H(t) - H(t-1)$ units of currency for goods, so that $H(t)$ is the total supply of currency at time- t , to be carried over by the young into period- $t+1$.

- (c) Show that if $1/z > N_2\beta/N_1\alpha$ there exists a continuum of equilibria with valued fiat currency.
- (d) Display the unique stationary monetary equilibrium, which takes the form of a “quantity theory” equation. Compute the equilibrium rate of return on currency holdings and consumption loans. Show that if $1/z < N_2\beta/N_1\alpha$, then a monetary equilibrium does not exist. (Hint: Look at the rate of return on consumption loans in the nonmonetary equilibrium).
- (e) Find the value of z that *maximizes* the government’s revenue from money creation. Compare this with the largest value of z that is compatible with the existence of a monetary equilibrium. Interpret the results.
5. This question uses data from the file **epdata.m** and the program **hanjanbnd.m**. Both can be downloaded from the class webpage.

Consider the following annual data for real gross returns on U.S. stocks and Treasury bills from 1890 to 1979 (these are the data originally used by Mehra and Prescott (1985)). The mean returns are $\mu = [1.07 \quad 1.02]$, respectively, and the covariance matrix of returns is

$$\begin{bmatrix} .0274 & .00104 \\ .00104 & .00308 \end{bmatrix}$$

- (a) For the data on *excess* returns of stocks over bonds, compute the Hansen-Jagannathan bound on the stochastic discount factor. Plot the bound as a function of $E(m)$ on the interval $[.9, 1.02]$.
- (b) Using data on raw returns (i.e, both stocks and bills), compute and plot the Hansen-Jagannathan bound on the same interval. Plot the bound on the same figure used in part (a).
- (c) Using the data in **epdata.m** and the program **hanjagbnd.m**, compute the mean and standard deviation for m_t for three different utility function specifications. In particular, assume $m_{t+1} = \beta u'(c_{t+1})/u'(c_t)$, where $u(c) = c^{1-\gamma}/(1-\gamma)$, and let $\beta = .99$ and $\gamma = 0, 5$ and 10 . Plot them on the same graph from part (b). Do the points lie within the Hansen-Jagannathan bounds? What do you conclude?