SIMON FRASER UNIVERSITY
Department of Economics

Econ 809 | Prof. Kasa
Advanced Macroeconomic Theory | Spring 2004

PROBLEM SET 4 - Insurance and Incentives
(Due April 8)

1. Consider an endowment economy populated by a large number of individuals with identical quadratic preferences

$$E\sum_{t=0}^{\infty}\beta^t u(c_t) = E\sum_{t=0}^{\infty}\beta^t\left(4c_t - \frac{c_t^2}{2}\right), \qquad \text{with } \beta = 0.8$$

There are two types of individuals. Type I individuals receive an endowment of 0 goods with probability 0.5 and an endowment of 2 goods with probability 0.5 in any given period. The endowments of Type II individuals are perfectly negatively correlated with the endowments of Type I, so that from the law of large numbers, the per capita endowment is constant and equal to 1 in all periods.

There is a social planner who attempts to provide insurance to the agents. The planner does *not* have access to outside funds. He simply reallocates the existing goods between the two types of individuals. He cares equally about all agents. However, the individuals are unable to commit to a risk-sharing contract, i.e., they are free to walk away from the social contract at any time, but they must then live in autarky from that point onwards.

(a) Compute the optimal insurace contract when the planner lacks memory, ie., when transfers in any given period can only depend on the current endowment realizations.

(b) Can the insurance contract in part (a) be improved if we allow for history-dependent transfers? Why or why not?

(c) Explain how the optimal contract changes when β goes to one. Explain how the optimal contract changes when β goes to zero.

2. An unemployed worker values stochastic sequences of consumption, c_t, and search effort, a_t, according to

$$E\sum_{t=0}^{\infty}\beta^t[u(c_t) - a_t]$$

where $\beta \in (0,1)$, $c_t \geq 0$, and $a_t \geq 0$. All jobs are alike and pay a wage $w > 0$ each period forever. Assume there is an unemployment insurance agency which provides a transfer during unemployment. As in Hopenhayn and Nicolini (1997), assume the insurer can also tax employed workers after they find a job. Assume the tax is lump sum at rate τ per period, so that an employed worker's consumption is $w - \tau$. The tax can be dependent on the worker's prior history of unemployment.

The probability of finding a job is $p(a)$, which is an increasing, strictly concave, and twice differentiable function of search effort, with $p(0) = 0$. The consumption good is nonstorable, individuals cannot borrow or lend, and they do not have any asset holdings. Hence, all consumption smoothing must occur via the unemployment insurance contract.

Finally, assume that the insurance agency *can* observe the worker's search effort and consumption.

(a) Let V_{aut} be the value of an unemployed worker's expected discounted utility when he has no access to unemployment insurance. The insurance agency wants to deliver discounted utility $V > V_{aut}$ to workers at minimum expected discounted cost $C(V)$. Assuming that the insurance agency has the same discount factor as workers, formulate the Bellman equation of the insurance agency. (Hint: Note that the ability to tax workers after employment implies that V^e is a control variable for the insurance agency, in addition to c, a, and V^u.)

(b) Prove that the optimal policy satisfies $c = w - \tau$. Interpret this condition.

3. Consider an unemployed person with preferences,

$$E \sum_{t=0}^{\infty} \beta^t u(c_t)$$

Each period an unemployed worker draws a single wage offer, w, from a uniform wage distribution on the interval $[w_L, w_H]$. Let the cumulative distribution function be denoted $F(x) = \text{prob}[w \leq x]$, and denote its (constant) density by f. After a worker has accepted an offer, he receives the wage w forever, ie, there is no risk of further unemployment. Also, in contrast to the previous question, now assume that employed workers are beyond the grasp of the unemployment insurance agency, ie., wages cannot be taxed. However, like the previous question, assume that workers do not hold assets and cannot borrow and lend outside the contract.

(a) Characterize the worker's optimal reservation wage when he is entitled to a time-invariant unemployment compensation b of indefinite duration.

(b) Characterize the optimal unemployment compensation policy under full information, ie., when the insurance agency can observe and control the worker's consumption and reservation wage.

(c) Now characterize the optimal unemployment compensation policy under asymmetric information, ie., when the insurance agency *cannot* observe wage offers, although it continues to observe and thereby control the worker's consumption. Be sure to discuss and interpret the optimal time path of an unemployed worker's consumption level.

4. A household orders sequences $\{c_t\}_{t=0}^{\infty}$ of a single nondurable good according to

$$E \sum_{t=0}^{\infty} \beta^t u(c_t) \qquad \beta \in (0,1)$$

where $u'(0) = +\infty$. Each period the household receives an endowment y_t that obeys a discrete state Markov chain with $P_{ij} = \text{Prob}(y_{t+1} = y_j | y_t = y_i)$, where the endowment y_t can take one of S values $[y_1, \cdots y_S]$.

Conditional on having observed the time-t realization of the household's endowment, a social insurer wants to deliver expected discounted utility v to the household at minimum cost. The insurer observes y_t at the beginning of every period, and contingent on the observed history of endowments, can make a transfer τ_t to the household. The transfer can be positive or negative and can be enforced without cost. Finally, let $C(v, i)$ denote the minimum cost of delivering promised discounted utility v when the household has just received endowment y_i.

(a) Assuming the insurer shares the same discount factor β, write down the Bellman equation for $C(v, i)$.

(b) Characterize the consumption and transfer policies that attain $C(v, i)$. Derive the associated law of motion for promised discounted utility v.

(c) Assume that the household is socially isolated, and has no access to insurance. Let $v_{aut}(i)$ be the expected discounted value of utility for a household in autarky, conditional on current income being y_i. Formulate the Bellman equations for $v_{aut}(i), i = 1, \cdots S$.

(d) Return to the problem of the insurer, but now assume the insurer *cannot* enforce transfers, since the household is free to walk away from the contract at any time and instead live in autarky. Hence, the insurer must now design a history-dependent transfer policy that prevents the household from exercising its option to walk away. Again letting $C(v, i)$ be the insurer's value function, formulate the insurer's Bellman equation. Briefly describe and interpret the implied law of motion for v.