SIMON FRASER UNIVERSITY
Department of Economics

Econ 808 | Prof. Kasa
Macroeconomic Theory | Fall 2010

PROBLEM SET 4 - Asset Pricing
(Due December 1)

1. Consider a complete markets economy with a representative agent. There is one good in the economy, which arrives as an exogenous endowment following the process:

$$y_{t+1} = \lambda_{t+1} y_t$$

where $y_t$ is the endowment at time-$t$ and $\{\lambda_{t+1}\}$ follows a two-state Markov chain with transition matrix

$$P = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix}$$

and initial distribution $\pi_\lambda = [\pi_0 \quad 1 - \pi_0]$. The value of $\lambda_t$ is given by $\bar{\lambda}_1 = 0.98$ in state 1 (the 'bad' state) and $\bar{\lambda}_2 = 1.03$ in state 2 (the 'good' state). Assume the history of $y_s, \lambda_s$ up to $t$ is observed at time-$t$. The consumer ranks consumption sequences according to the utility function $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$ where $\beta \in (0,1)$ and $u(c) = c^{1-\gamma}/(1-\gamma)$, where $\gamma \geq 1$.

   **(a)** Carefully define a competitive equilibrium, and describe how to compute it.

   In what follows, assume $p_{11} = .8$, $p_{22} = .85$, $\pi_0 = .5$, $\beta = .96$, and $\gamma = 2$. Suppose the economy begins with $\lambda_0 = .98$ and $y_0 = 1$.

   **(b)** Compute the unconditional average growth rate of consumption (i.e., before observing $\lambda_0$).

   **(c)** Compute the time-0 prices of three risk-free discount bonds, i.e., those promising to pay one unit of time-$j$ consumption for $j = 0, 1, 2$, respectively.

   **(d)** Compute the time-0 prices of three state-contingent bonds, i.e., those promising to pay one unit of time-$j$ consumption contingent on $\lambda_j = \bar{\lambda}_1$ for $j = 0, 1, 2$.

   **(e)** Compute the time-0 prices of three state-contingent bonds, only this time assume they are contingent on state 2, again at dates $j = 0, 1, 2$.

   **(f)** Compare the results from parts c, d, and e. Interpret the results.

2. This question uses data from the file **epdata.m** and the program **hanjanbnd.m**. Both can be downloaded from the class webpage.

   Consider the following annual data for real gross returns on U.S. stocks and Treasury bills from 1890 to 1979 (these are the data originally used by Mehra and Prescott

(1985)). The mean returns are $\mu = [1.07 \quad 1.02]$, respectively, and the covariance matrix of returns is

$$\begin{bmatrix} .0274 & .00104 \\ .00104 & .00308 \end{bmatrix}$$

**(a)** For the data on *excess* returns of stocks over bonds, compute the Hansen-Jagannathan bound on the stochastic discount factor. Plot the bound as a function of $E(m)$ on the interval $[.9, 1.02]$.

**(b)** Using data on raw returns (i.e, both stocks and bills), compute and plot the Hansen-Jagannathan bound on the same interval. Plot the bound on the same figure used in part (a).

**(c)** Using the data in **epdata.m** and the program **hanjagbnd.m**, compute the mean and standard deviation for $m_t$ for three different utility function specifications. In particular, assume $m_{t+1} = \beta u'(c_{t+1})/u'(c_t)$, where $u(c) = c^{1-\gamma}/(1-\gamma)$, and let $\beta = .99$ and $\gamma = 0, 5$ and 10. Plot them on the same graph from part (b). Do the points lie within the Hansen-Jagannathan bounds? What do you conclude?

3. Suppose the only assets in the economy are infinitely lived trees. Output equals the fruit of the trees, which is exogenous and nonstorable. Thus, $C_t = Y_t$, where $Y_t$ is the exogenously determined per capita output, and $C_t$ is per capita consumption. Assume that initially each agent owns the same number of trees. Note that since agents are assumed to be identical, the equilibrium price of a tree must be such that each agent does not wish to either increase or decease his or her holdings of the tree.

   Let $P_t$ denote the price of a tree in period-$t$. Assume that if a tree is sold, the sale occurs after the original owner receives that period's output (ie, prices are 'ex-dividend'). Finally, assume that the representative agent maximizes,

$$E_t \sum_{j=0}^{\infty} \beta^j \ln C_{t+j}$$

**(a)** Write down the Euler equation for asset prices.

**(b)** Assume that $\lim_{s\to\infty} E_t[\beta^s(P_{t+s}/Y_{t+s})] = 0$. Given this assumption, iterate your answer to part (a) forward to solve for $P_t$. (Hint: Impose the equilibrium condition $C_{t+j} = Y_{t+j}$ for all $j$.)

**(c)** Explain intuitively why an increase in expected future dividends does not affect asset prices. (Hint: Think in terms of income and substitution effects.)

**(d)** Note that in general consumption does not follow a random walk in this model. Why not? What's the key difference between this model and Hall's (JPE, 1978) model?