

SIMON FRASER UNIVERSITY  
Department of Economics

Econ 808  
Macroeconomic Theory

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PROBLEM SET 5 - Fiscal Policy  
(Due December 2)

1. Consider an economy populated by a government and a representative household. Both have infinite horizons. There is no uncertainty. The government consumes a constant amount  $g_t = \bar{g}, t \geq 0$  of the single consumption good. The government also sets sequences for two types of taxes,  $\{\tau_{ct}, \tau_{ht}\}_{t=0}^{\infty}$ , where  $\tau_{ct}$  is flat-rate tax on consumption and  $\tau_{ht}$  is a lump-sum 'head tax'. Household preferences are given by

$$\max_{c_t} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where  $u(\cdot)$  is increasing, strictly concave, and continuously differentiable. The economy's aggregate resource constraint is given by

$$g_t + c_t + k_{t+1} \leq f(k_t) + (1 - \delta)k_t$$

where  $\delta$  is the depreciation rate of capital.

At time 0, there are complete markets for dated commodities, and the household faces the budget constraint

$$\sum_{t=0}^{\infty} \{q_t[(1 + \tau_{ct})c_t + k_{t+1} - (1 - \delta)k_t]\} \leq \sum_{t=0}^{\infty} \{r_t k_t + w_t - q_t \tau_{ht}\}$$

where  $q_t$  is the time-0 price of time- $t$  consumption goods,  $r_t$  is the time-0 rental rate of date- $t$  capital, and  $w_t$  is the wage rate for date- $t$  labor. Note that capital is neither taxed nor subsidized.

Output is produced by competitive profit-maximizing firms, who choose factor inputs to maximize

$$\sum_{t=0}^{\infty} [q_t f(k_t) n_t - w_t n_t - r_t k_t n_t]$$

where  $k_t$  is the firm's capital/labor ratio, and  $n_t$  is labor input. (Note: given preferences, labor is supplied inelastically).

Finally, the government chooses taxes to finance its constant expenditure path,

$$\sum_{t=0}^{\infty} q_t (c_t \tau_{ct} + \tau_{ht}) = \sum_{t=0}^{\infty} q_t \bar{g}$$

- (a) Carefully define a competitive equilibrium.
- (b) Suppose historically the government had access to lump-sum taxes, and used them exclusively to finance expenditure. Characterize the steady-state capital/labor ratio for this economy.
- (c) For the economy described in part (b), prove that Ricardian Equivalence holds (i.e., prove that the timing of taxes is irrelevant).
- (d) Let  $\bar{k}_0$  be the steady state value of  $k_t$  that you found in part (b), and let this be the initial value of capital for the following experiment. Assume that suddenly and unexpectedly, a court decision rules that lump-sum taxes are unconstitutional, and that starting from now on the government must finance expenditures using consumption taxes,  $\tau_{ct}$ . The value of  $g_t$  remains constant at  $\bar{g}$ .  
 Policy advisor #1 proposes the following tax policy: find the *constant* consumption tax that satisfies the government budget constraint, and impose it immediately and keep it there. Compute the new steady state  $k_t$  under this policy. Describe the transition path.
- (e) Policy advisor #2 proposes the following alternative tax policy. Instead of suddenly imposing the increase in  $\tau_{ct}$ , he suggests ‘easing the pain’ by postponing the increase for 10 years. That is, he proposes setting  $\tau_{ct} = 0$  for  $t = 0, 1, \dots, 9$ , and then setting  $\tau_{ct} = \bar{\tau}_c$  for  $t \geq 10$ , where  $\bar{\tau}_c$  is selected to satisfy the time-0 government budget constraint. Compute the steady-state  $k_t$  associated with this policy. Describe the transition path.
- (f) Whose advice should you follow? Why?
2. Consider the nonstochastic model discussed in the first part of chapter 15, only now assume the economy is a small open economy that cannot affect the international rental rate on capital,  $r_t^*$ . Domestic firms can rent any amount of capital at this price, and households and the government can choose to go short or long in the international capital market at this rental price. That is, capital is completely mobile internationally. However, labor is *not* mobile internationally. Assume the government levies a tax  $\tau_{nt}$  on labor income, but does not levy any tax on capital income. Instead, assume the government levies a tax  $\hat{\tau}_t^k$  on domestic firms’ rental payments to capital, *regardless of the capital’s country of origin* (domestic or foreign). Thus, domestic firms face a total cost of  $(1 + \hat{\tau}_t^k)r_t^*$  on each unit of rented capital in period- $t$ .
- (a) Solve for the optimal capital tax  $\hat{\tau}_t^k$  (i.e., solve the Ramsey problem).
- (b) Compare the optimal tax policy for this small open economy to what you would get for a closed economy.
3. Consider the following nonstochastic optimal taxation problem. A government must finance an exogenous stream of expenditures using only labor and consumption taxes,  $\{\tau_{nt}, \tau_{ct}\}$ . As a result, the household’s budget constraint becomes

$$\sum_{t=0}^{\infty} q_t(1 + \tau_{ct})c_t \leq \sum_{t=0}^{\infty} q_t(1 - \tau_{nt})w_t n_t + [r_0 + 1 - \delta]k_0 + b_0$$

where again  $q_t$  is the time-0 price of time- $t$  consumption.

- (a) Characterize the Ramsey tax plan. In particular, describe the limiting sequence of consumption taxes.
- (b) In the case of capital taxation, discussed in chapter 15, an exogenous upper bound on  $\tau_0^k$  was imposed. Why? Explain why a similar exogenous bound on  $\tau_0^c$  is needed to ensure a nondegenerate Ramsey problem. (Hint: Explore the implications of setting  $\tau_{ct} = \bar{\tau}_c$  and  $\tau_{nt} = -\bar{\tau}_c$  for all  $t \geq 0$ , where  $\bar{\tau}_c$  is a large positive number.)