*Rational Expectations and the Reconstruction of Macroeconomics**

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Fans of the National Football League may well have observed the following behavior by the Houston Oilers during the 19— season. At home against Kansas City, when confronted with a fourth down in its own end of the field, Houston punted 100 percent of the time. The next week, at St. Louis, in the same situation, Houston punted 93 percent of the time. The following week at Oakland, again in that situation, Houston again punted 100 percent of the time, as it did the subsequent week at home against San Diego, and so on and on for the rest of the season. In short, on the basis of the time series data, Houston has a tendency to punt on fourth downs in its own territory, no matter what team it plays or where.

Having observed this historical record, suppose it is our task to predict how Houston will behave in the future on fourth and long in its own territory. For example, suppose that next week Houston is to play an expansion team at Portland that it has never played before. It seems safe to predict that Houston will punt on fourth downs in its own territory at Portland. This sensible prediction is not based on any understanding of the game of football, but rather on simply extrapolating a past behavior pattern into the future.

In many cases, we would expect this method of prediction to work well. However, for precisely those cases in which predictions are most interesting, the extrapolative method can be expected to break down. For instance, suppose that the Commissioner of the National Football League announced a rule change, effective next Sunday, which gave a team six downs in which to make a first down. Would we still expect Houston to punt on fourth down? Clearly not; at least no one familiar with the game of football would.

What this example indicates is that historical patterns of human behavior often depend on the rules of the game in which people are participating. Since much human behavior is purposeful, it makes sense to expect that it will change to take advantage of changes in the rules. This principle is so familiar to fans of football and other sports that it hardly bears mentioning. However, the principle very much deserves mentioning in the context of economic policy because here it has been routinely ignored—and with some devastating results.[1] Adherents of the theory of rational expectations believe, in fact, that no less than the field of macroeconomics must be reconstructed in order to take account of this principle of human behavior. Their efforts to do that involve basic changes in the ways economists formulate, simulate, and predict with econometric models. They also call for substantial changes in the ways economic policymakers frame their options.[2]

Models must let behavior change with the rules of the game

In order to provide quantitative advice about the

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[1]Charles Whiteman and Ian Bain are responsible for impressing upon me the many parallels between football and macroeconomics.

[2]This is the message of Lucas 1976.

effects of alternative economic policies, economists have constructed collections of equations known as *econometric models*.[3] For the most part, these models consist of equations that attempt to describe the behavior of economic agents—firms, consumers, and governments—in terms of variables which are assumed to be closely related to their situations. Such equations are often called *decision rules* since they describe the decisions people make about things like consumption rates, investment rates, and portfolios as functions of variables that summarize the information people use to make those decisions. For all of their mathematical sophistication, econometric models amount to statistical devices for organizing and detecting patterns in the past behavior of people's decision making, patterns which can then be used as a basis for predicting their future behavior.

As devices for extrapolating future behavior from the past under a given set of rules of the game, or government policies, these models appear to have performed well.[4] They have not, however, when the rules have changed. In formulating advice for policymakers, economists have routinely used these models to predict the consequences of historically unprecedented, hypothetical government interventions that can only be described as changes in the rules of the game. In effect, the models have been manipulated in a way which amounts to assuming that people's patterns of behavior do not depend on those properties of the environment that government interventions would change. The assumption has been, that is, that people will act under the new rules just as they have under the old, so that even under new rules past behavior is still a reliable guide to future behavior. Econometric models used in this way have not been able to accurately predict the consequences of historically unparalleled interventions.[5] To take one painful recent example, standard Keynesian and monetarist econometric models built in the late 1960s failed to predict the effects on output, employment, and prices that were associated with the unprecedented large deficits and rates of money creation of the 1970s.

Recent research has been directed at building econometric models that take into account that people's behavior patterns will vary systematically with changes in government policies or the rules of the game.[6] Most of this research has been conducted by adherents of the so-called hypothesis of rational expectations. They model people as making decisions in dynamic settings in the face of well-defined constraints. Included among these constraints are laws of motion over time that describe such things as the taxes that people must pay and the prices of the goods that they buy and sell. The hypothesis of rational expectations is that people understand these laws of motion. The aim of the research is to build models that can predict how people's behavior will change when they are confronted with well-understood changes in ways of administering taxes, government purchases, aspects of monetary policy, and the like.

The Investment Decision as an Example

A simple example will serve to illustrate both the principle that decision rules depend on the laws of motion that agents face and the extent that standard macroeconomic models have violated this principle. Let k_t be the capital stock of an industry and τ_t be a tax rate on capital. Let τ_t be the first element of z_t, a vector of current and lagged variables including those that the government considers when it sets the tax rate on capital. We have $\tau_t \equiv e^T z_t$, where e is the unit vector with unity in the first place and zeros elsewhere.[7] Let a firm's optimal accumulation plan require that capital acquisitions obey[8]

$$k_t = \lambda k_{t-1} - \alpha \sum_{j=0}^{\infty} \delta^j E_t \tau_{t+j} \qquad \begin{array}{c} \alpha > 0 \\ 0 < \lambda < 1 \\ 0 < \delta < 1 \end{array} \tag{1}$$

[3] Lucas and Sargent 1979 provides a brief explanation of econometric models and their uses in macroeconomics.

[4] This evidence is cited by Litterman (1979) and his references.

[5] Sims (1980) and Lucas (1976) describe why econometric models can perform well in extrapolating the future from the past, assuming no changes in rules of the game, while performing poorly in predicting the consequences of changes in the rules.

[6] For an example of such research and extensive lists of further references, see Hansen and Sargent 1980 and Lucas and Sargent forthcoming.

[7] Here T denotes matrix transposition.

[8] The investment schedule (1) can be derived from the following dynamic model of a firm. A firm chooses sequences of capital to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ f_1 k_t - \frac{f_2}{2} k_t^2 - f_3 k_t \tau_t - \frac{d}{2}(k_t - k_{t-1})^2 \right\} \tag{*}$$

where $f_1, f_2, f_3, d > 0$; $0 > \beta > 1$; and E_0 is the mathematical expectation operator conditioned on information known at time 0. The maximization is subject to (k_{t-1}, τ_t) being known at the time t. Maximization problems of this kind are analyzed in Sargent 1979. The parameters λ, α, and δ can be shown to be functions of f_1, f_2, f_3, and d.

where $E_t\tau_{t+j}$ is the tax rate at time t which is expected to prevail at time $(t+j)$.

Equation (1) captures the notion that the demand for capital responds negatively to current and future tax rates. However, equation (1) does not become an operational investment schedule or decision rule until we specify how agents' views about the future, $E_t\tau_{t+j}$, are formed. Let us suppose that the actual law of motion for z_t is

$$(2) \qquad z_{t+1} = Az_t$$

where A is a matrix conformable with z_t.[9] If agents understand this law of motion for z_t, the first element of which is τ_t, then their best forecast of τ_{t+j} is $e^T A^j z_t$. We impose rational expectations by equating agents' expectations $E_t\tau_{t+j}$ to this best forecast. Upon imposing rational expectations, some algebraic manipulation implies the operational investment schedule

$$(3) \qquad k_t = \lambda k_{t-1} - \alpha e^T (I - \delta A)^{-1} z_t.$$

In terms of the list of variables on the right-hand side, equation (3) resembles versions of investment schedules which were fit in the heyday of Keynesian macroeconomics in the 1960s. This is not unusual, for the innovation of rational expectations reasoning is much more in the ways equations are interpreted and manipulated to make statements about economic policy than in the look of the equations that are fit. Indeed, the similarity of standard and rational expectations equations suggests what can be shown to be true generally: that the rational expectations reconstruction of macroeconomics is not mainly directed at improving the statistical fits of Keynesian or monetarist macroeconomic models over given historical periods and that its success or failure cannot be judged by comparing the R^2's of reconstructed macroeconomic models with those of models constructed and interpreted along earlier lines.

Under the rational expectations assumption, the investment schedule (3) and the laws of motion for the tax rate and the variables that help predict it (2) have a common set of parameters, namely, those of the matrix A. These parameters appear in the investment schedule because they influence agents' expectations of how future tax rates will affect capital. Further, notice that all of the variables in z_t appear in the investment schedule, since via equation (2) all of these variables help agents forecast future tax rates. (Compare this with the common econometric practice of using only current and lagged values of the tax rate as proxies for expected future tax rates.)

The fact that (2) and (3) share a common set of parameters (the A matrix) reflects the principle that firms' optimal decision rule for accumulating capital, described as a function of current and lagged state and information variables, will depend on the constraints (or laws of motion) that firms face. That is, the firm's pattern of investment behavior will respond systematically to the rules of the game for setting the tax rate τ_t. A widely understood change in the policy for administering the tax rate can be represented as a change in the first row of the A matrix. Any such change in the tax rate regime or policy will thus result in a change in the investment schedule (3). The dependence of the coefficients of the investment schedule on the environmental parameters in A is reasonable and readily explicable as a reflection of the principle that agents' rules of behavior change when they encounter changes in the environment in the form of new laws of motion for variables that constrain them.

To illustrate this point, consider two specific tax rate policies. First consider the policy of a *constant* tax rate $\tau_{t+j} = \tau_t$ for all $j \geq 0$. Then $z_t = \tau_t$, $A = 1$, and the investment schedule is

$$(4) \qquad k_t = \lambda k_{t-1} + h_0\tau_t \qquad \text{where } h_0 = -\alpha/(1-\delta).$$

Now consider an *on-again, off-again* tax rate policy of the form $\tau_t = -\tau_{t-1}$. In this case $z_t = \tau_t$, $A = -1$, and the investment schedule becomes

$$(5) \qquad k_t = \lambda k_{t-1} + h_0\tau_t \qquad \text{where now } h_0 = -\alpha/(1+\delta).$$

Here the investment schedule itself changes as the policy for setting the tax rate changes.

Standard econometric practice has not acknowledged that this sort of thing happens. Returning to the more general investment example, the usual econometric practice has been roughly as follows. First, a model is typically specified and estimated of the form

[9] The eigenvalues of A are assumed to be less than δ^{-1} in absolute value.

$$(6) \qquad k_t = \lambda k_{t-1} + hz_t$$

where h is a vector of free parameters of dimension conformable with the vector z_t. Second, holding the parameters h fixed, equation (6) is used to predict the implications of alternative paths for the tax rate τ_t. This procedure is equivalent to estimating equation (4) from historical data when $\tau_t = \tau_{t-1}$ and then using this same equation to predict the consequences for capital accumulation of instituting an on-again, off-again tax rate policy of the form $\tau_t = -\tau_{t-1}$. Doing this assumes that a single investment schedule of the form (6) can be found with a single parameter vector h that will remain fixed regardless of the rules for administering the tax rate.[10]

The fact that equations (2) and (3) share a common set of parameters implies that the search for such a regime-independent decision schedule is misdirected and bound to fail. This theoretical presumption is backed up by the distressing variety of instances in which estimated econometric models have failed tests for stability of coefficients when new data are added. This problem cannot be overcome by adopting more sophisticated and more general lag distributions for the vector h, as perhaps was hoped in the 1960s.

General Implications

The investment example illustrates the general presumption that the systematic behavior of private agents and the random behavior of market outcomes both will change whenever agents' constraints change, as when government policy or other parts of the environment change. To make reliable statements about policy interventions, we need dynamic models and econometric procedures which are consistent with this general presumption. Foremost, we need a new and stricter definition of the class of parameters that can be regarded as *structural*. The body of doctrine associated with the simultaneous equations model in econometrics properly directs the attention of the researcher beyond reduced form parameters to the parameters of structural equations which are meant to describe those aspects of people's behavior that remain constant across a range of hypothetical environments. Although such structural parameters are needed to analyze an interesting class of policy interventions, most often included among them have been parameters of equations describing the rules of choice for private agents. Consumption functions, investment schedules, and demand functions for assets are all examples of such rules of choice. In dynamic settings, regarding the parameters of these rules of choice as structural or invariant under policy interventions violates the principle that optimal decision rules depend on the environment in which agents believe they are operating.

If parameters of decision rules cannot be regarded as structural or invariant under policy interventions, deeper objects that can must be sought. The best that can be hoped for is that parameters characterizing private agents' preferences and technologies will not change when changes in economic policy change the environment. If dynamic econometric models were formulated explicitly in terms of the parameters of preferences, technologies, and constraints, in principle they could be used to predict the effects on observed behavior of changes in policy rules. In terms of our investment example with equations (2) and (3), the idea would be to estimate the free parameters of the model (λ, α, δ, A). With these estimates, economists could predict how the investment schedule would change if different A's occurred.[11]

Policymakers must choose among alternative rules, not isolated actions

These ideas have implications not only for theoretical and econometric practices, but also for the ways in which policymakers and their advisers think about the choices confronting them. In particular, the rational expectations approach directs attention away from particular isolated actions and toward choices among feasible rules of the game, or repeated strategies for choosing policy variables. While Keynesian and monetarist macroeconomic models have been used to try to analyze what the effects of isolated actions would be, it is now clear that the answers they have given have necessarily been bad, if only because such questions are ill-posed.

In terms of our investment example, by selecting different values for the first row of A, we can analyze the effects on current and subsequent investment of switching from one well-understood policy for setting

[10] This is analogous to assuming that Houston's propensity to punt on fourth down does not depend on the number of downs per series determined by the NFL rules.

[11] As claimed in footnote 8, the parameters (λ, α, δ) can be shown to be functions of the parameters (f_1, f_2, f_3, d) of the present value function being maximized in (*).

the tax rate to another—that is, we can analyze the effects of different *strategies* for setting the tax rate. However, we cannot analyze the effects on current and subsequent investment of alternative *actions* consisting of different possible settings for the tax rate τ_t at a particular point in time $t = \bar{t}$. For in order to make predictions, we must specify agents' views about the law of motion A, and this is not done when we simply consider actions consisting of alternative settings for τ_t at one isolated point in time. This idea is so widely accepted as to be uncontroversial among decision theorists (and football fans); but even today practicing macroeconomists usually ignore it.

To take a concrete example, in the United States there was recently interest in analyzing what would happen to the rate of domestic extraction of oil and gas if the tax on profits of oil producers increased a lot on a particular date. Would supply go up or down if the tax were raised to X percent on July 1? The only scientifically respectable answer to this question is "I don't know." Such a rise in the oil-profits tax rate could be interpreted as reflecting one of a variety of different tax strategies (A matrices), each with different implications for current and prospective extraction of oil.

For example, suppose that oil companies had reason to believe that the increase in the tax is temporary and will be repealed after the election. In that case, they would respond by decreasing their rate of supply now and increasing it later, thus reallocating their sales to periods in which their shareholders get a larger share of profits and the government a smaller share. Yet suppose that oil companies believed that the increase in the tax rate on July 1 is only the beginning and that further increases will follow. In that case, the response to the tax rate increase would be the reverse: to increase supply now and decrease it later in order to benefit companies' shareholders. This example illustrates that people's views about the government's strategy for setting the tax rate are decisive in determining their responses to any given actions and that the effects of actions cannot be reliably evaluated in isolation from the policy rule or strategy of which they are an element.

What policymakers (and econometricians) should recognize, then, is that societies face a meaningful set of choices about alternative economic policy regimes. For example, the proper question is not about the size of tax cut to impose now in response to a recession, but about the proper strategy for repeatedly adjusting tax rates in response to the state of the economy, year in and year out. Strategic questions of this nature abound in fiscal, monetary, regulatory, and labor market matters. Private agents face the problem of determining the government regime under which they are operating, and they often devote considerable resources to doing so. Whether governments realize it or not, they do make decisions about these regimes. They would be wise to face these decisions deliberately rather than ignore them and pretend to be able to make good decisions by taking one seemingly unrelated action after another.

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