Angular Kinetics

Hamill & Knutzen (Ch 11)
Hay (Ch. 6), Hay & Ried (Ch. 12), Kreighbaum & Barthels (Module I & J) or Hall (Ch. 13 & 14)

Components of Torque
(review from “Systems FBD” lecture)

- axis of rotation (fulcrum)
- force (not directed through axis of rotation)
- force (moment) arm

\[ \tau = F \times d \]

Muscles Create Torques

Although human motion is general (translation and rotation), it is generated by a series of torques and hence rotations.

The line of action of muscle forces do not pass through the joint axis of rotation

Torque is a Vector

- Torque has both magnitude (force x force arm) and direction.
- A counter clockwise torque is positive and a clockwise torque is negative.
- Make sure that you know which direction the torque is being applied

Work versus Torque

- Work is a scalar
- \( F \times d = MLT^{-2} \times L = ML^2T^{-2} \)
- As the L’s are the same, the square of them will always be positive.
- Dot product of vectors
- Torque is a vector
- \( F \times d_{\text{perpendicular}} = MLT^{-2} \times L \)
- As the L’s are perpendicular to each other one could be positive the other negative, therefore torque has direction.
- Cross product of vectors

Force Arm is Perpendicular to Force Line of Action

Make sure you use the force arm not the distance from point of force application to axis of rotation.
Total Body Centre of Mass

- This is not a difficult concept but it can be very time consuming.
- If you know the locations of the segment centre of masses, you can calculate the total body centre of mass.
- “The sum of the torques equals the torque of the sum”.

\[ M_Tg_T = m_1g_x_1 + m_2g_x_2 + m_3g_x_3 \]

Angular Equivalents

- Moment of force (torque) can be thought of as the angular equivalent of force.
- There are numerous angular equivalents that you should be aware.
- Don’t treat Angular Kinetics as something entirely separate from Linear Kinetics.

Segment Inertial Properties

- Mass, length and centre of mass location is sufficient for static analysis of the forces and moments at each joint for any given body posture.
- Because the segments rotate during dynamic activities, the moment of inertia (I) of the body segments must be known.
- Do not worry about how I is calculated but you must understand what it is!

\[ I = \sum_{i=1}^{n} m_i r_i^2 \]
Whole Leg Moment of Inertia

<table>
<thead>
<tr>
<th>Moment of Inertia (kg.m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
</tr>
<tr>
<td>2.5</td>
</tr>
<tr>
<td>2.0</td>
</tr>
<tr>
<td>1.5</td>
</tr>
<tr>
<td>1.0</td>
</tr>
</tbody>
</table>

Leg Position

<table>
<thead>
<tr>
<th>Moment of Inertia (kg.m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
</tr>
<tr>
<td>2.5</td>
</tr>
<tr>
<td>2.0</td>
</tr>
<tr>
<td>1.5</td>
</tr>
<tr>
<td>1.0</td>
</tr>
</tbody>
</table>

Whole Leg Moment of Inertia

Moment of Inertia

Radius of Gyration

The angular equivalent of centre of mass.

That distance at which all of the body’s mass can be said to act.

<table>
<thead>
<tr>
<th>Radii of Gyration / segment length (transverse axis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment</td>
</tr>
<tr>
<td>head, neck, trunk</td>
</tr>
<tr>
<td>upper arm</td>
</tr>
<tr>
<td>arm</td>
</tr>
<tr>
<td>hand</td>
</tr>
<tr>
<td>thigh</td>
</tr>
<tr>
<td>leg</td>
</tr>
<tr>
<td>foot</td>
</tr>
</tbody>
</table>

Questions

1. If the leg (shank) of a subject has a mass of 3.6 kg and is 0.4 m long. What is Ig?
2. Why is the % value in the table usually higher for the distal end than the proximal end?

Answers

1. Value from table (a) = k/L Therefore k = a x L
   Ig = mk² = 3.6 x (0.302 x 0.4)² = 0.0525 kg-m²
2. Muscle mass distribution is usually closer to proximal joint (speed of movement).
Parallel Axis Theorem

$$I_{\text{prox}} = I_g + mr^2$$

Moments of inertia in some typical diving and gymnastic positions
From: J.G. Hay 1993

We will look at airborne activities later

Angular Momentum

$$H = I\omega$$

Conservation of Angular Momentum
When gravity is the only external force acting on an object (projectile with no air resistance) then H is constant

How does a figure skater spin so fast?

Angular Equivalents

- Moment of force (torque) can be thought of as the angular equivalent of force.
- There are numerous angular equivalents that you should be aware.
- Don’t treat Angular Kinetics as something entirely separate from Linear Kinetics.
Newton’s First Law
“Every body continues in its state of rest or motion (constant velocity) in a straight line unless compelled to change that state by external forces exerted upon it.”

Rotational Analogue
“Every body continues to turn about its axis of rotation with constant angular momentum unless an external moment (torque) (eccentric force or couple) is exerted upon it.”

Newton’s Second Law
The rate of change of momentum of a body (or the acceleration for a body of constant mass) is proportional to the force causing it and the change takes place in the direction in which the force acts.

Rotational Analogue
The rate of change of angular momentum of a body is proportional to the moment (torque) causing it and the change takes place in the direction in which the moment (torque) acts.

\[ F = ma \]
\[ Ft = m\Delta v \]
\[ \tau = I\alpha \]
\[ \tau t = \Delta I\omega \]

Angular Impulse
The longer you can apply torque or the greater the torque you can apply over the same time period the greater the angular velocity.

Work-Energy Relationship
If force is applied off centre (i.e. the line of action of the force vector does not pass through the centre of rotation of the body) then rotation as well as translation will occur.

Angular Kinetics
\[ Work = \Delta Energy \]
\[ Fd = \Delta \frac{1}{2} mv^2 \]
\[ \tau\theta = \Delta \frac{1}{2} I\omega^2 \]

Linear Force?
\[ Fd = \Delta (\frac{1}{2} mv^2 + mgh) \]

Note: net force would have to factor in gravity.
Translation

\[ F_d = \Delta(\frac{1}{2}mv^2 + mgh) \]

Eccentric Force?

Rotation and Translation

This is because:

- The centre of mass of a rigid body instantaneously accelerates in the direction of the applied resultant external force, irrespective of where the force is applied on the body.
- The rotational effect is also instantaneous.
- However, you do not get something for nothing.

Rotation and Translation

You can break out the linear and rotational components of the eccentric force

\[ Work = \Delta \text{Energy} \]
\[ F_d = \Delta \frac{1}{2}mv^2 \]
\[ \tau \theta = \Delta \frac{1}{2}I \omega^2 \]

Tennis Anyone?

Blue vector (action on C of g)

\[ F_d = \Delta(\frac{1}{2}mv^2 + mgh) \]

Green vector

\[ \tau \theta = \Delta I \omega^2 \]

Racket
Newton’s Third Law
For every force applied by one body on a second, the second body applies an equal and oppositely directed force on the first.

Rotational Analogue
For every moment (torque) applied by one body on a second, the second body applies an equal and oppositely directed moment (torque) on the first.

A Review of Static Equilibrium
- If a system is stationary then the net external forces & external torques must equal zero (this is also true if the system is moving with constant velocities).
- The system is said to be in static equilibrium and we can often perform an analysis to find unknown torques and/or forces.
Rotation & Leverage

- All lever systems have:
  - An effort force (force)
  - A resistive force (resistance), and
  - An axis of rotation.

Mechanical Advantage (MA)

\[
MA = \frac{force \ arm}{resistance \ arm}
\]

Classes of Levers

1st Class: MA varies

2nd Class (MA > 1):
- Favours the effort force (i.e., a smaller effort force can balance a larger resistive force).

3rd Class (MA < 1):
- Favours range and speed of movement.

Third Class Levers

- The majority of musculoskeletal systems are in third class levers.
- These favour speed and range of movement.

Wheel & Axle Arrangements

\[
MA = \frac{radius \ of \ axle}{radius \ of \ wheel}
\]

- Only if the effort force is applied to the axle. If the effort force is on the wheel (resistive on the axis) reverse this fraction.
- Nearly all musculoskeletal systems in the human body are third class levers.
- Systems like rotator cuff muscles and other muscles responsible for longitudinal rotation of long bones can have MA’s <1. However, these MA’s are often quite close to 1.
Static Equilibrium & the C of G

- If a system is stationary then the net external forces & external torques must equal zero (this is also true if the system is moving with constant velocities).

What was that Kin 142 lab with the reaction board about?

Static Equilibrium

- Scale Reading = $R_1$ (multiple by $g$)
- Length of Board = 2 m
- Sum of torques about $A$ = zero
  
  \[ R_1 \times 2 = mg_{\text{board}} \times b \]

Scale Reading = $R_1$

\[ mg_{\text{board}} \]

\[ b \]

\[ A \]

Scale Reading = $R_2$

\[ mg_{\text{board}} \]

\[ b \]

\[ A \]

But: \[ R_1 \times 2 = mg_{\text{board}} \times b \]

Scale Reading = $R_2$

\[ (R_2 - R_1)2 = mg_{\text{subject}} \times X \]

Reaction Board Method

- Scale Reading = $R_1$
- Length of Board = 2 m

\[ mg_{\text{subject}} \]

\[ X \]
Static Equilibrium

\[ \Sigma F_x = 0 \]
\[ \Sigma F_y = 0 \]
\[ \Sigma M = 0 \]

Muscle Torque

If the mass of the forearm/hand segment and the location of its CG are known, plus similar information for any external forces (loads) we can calculate muscle torque.

What other information do we need if we want to calculate muscle force?

Line of action of forces in relation to segment (insertion point and angle of pull).

Static Equilibrium

Calculate the Muscle Force \( F_m \)?

\[ \Sigma F_x = 0 \]
\[ \Sigma F_y = 0 \]
\[ \Sigma M = 0 \]

Moment arms = 0.03m, 0.15m and 0.3 m respectively

Free Body Diagrams

- The point made on the last slide shows how the ability to identify the system and draw a free body diagram is an ESSENTIAL ability.
- We must use sum of moments about the elbow axis. This way we eliminate \( F_R \) from the equation as \( F_R \) passes through the elbow joint and hence, does not create a moment of force.

Static Equilibrium

Remember you have 2 unknown forces so you cannot directly solve for \( F_m \) using \( \Sigma F_y = 0 \).

Static Equilibrium

This was the result of summing the moments about the elbow.

\[ \Sigma F_x = 0 \]
\[ \Sigma F_y = 0 \]
\[ \Sigma M = 0 \]

Moment arms = 0.03m, 0.15m and 0.3 m respectively
Static Equilibrium
Calculate the joint reaction force ($F_R$)?

\[
\begin{align*}
\Sigma F_x &= 0 \\
\Sigma F_y &= 0 \\
\Sigma M &= 0
\end{align*}
\]

Simplification #1
Rigid Segments
- Note that we are assuming that the segments are rigid structures in these problems.
- From the skeletal lecture you will know that this is not exact.
- It is a good approximation however.

Simplification #2
Single Equivalent Muscles
- We have assumed there was a muscle producing the moment (or stabilizing the joint).
- However, this is obviously not accurate.
- For shoulder flexion for example we have two prime movers and two assistors.
- We often lump such muscle groups together and term them a “single equivalent muscle”.
- This may appear to be a gross simplification but you will see we do not make this assumption at many joints (spine, knee?)

Muscle-Joint Complexity
- Another important factor is whether the muscles crossing the joint have a common tendon, or similar insertion points and similar lines of action.
- Clearly many muscle joint systems are too complex to model in Kin 201.