1. a) Impulse equals change in momentum. \[ F_t = m(v_f - v_i) \]
Therefore \[ 25 = 0.8(V_f - 2) \]
As \[ V_i = 2 \]

Release speed of javelin = \( 33.25 \) m/s

I asked for speed as no information is provided for you to be able to calculate direction.

b) Total mechanical energy = kinetic energy + potential energy
\[ = \frac{1}{2}mv^2 + mgh \]
\[ = (0.5)(0.8)(33.25^2) + (0.8)(-9.81)(-1.6) \]
\[ = 442.225 + 12.557 \]

Mechanical energy of javelin at release = 455 Joules

c) The work done (Fd) on the javelin by the athlete equals the change in mechanical energy (KE & PE) of the javelin.

The energy at the start of the final throwing phase is the potential energy (mgh) and kinetic energy \( \frac{1}{2}mv^2 \).

\[ h = -1 \text{ m} \text{ and } m = 0.8 \text{ kg} \]
\[ v = 2 \text{ m/s} \text{ and } m = 0.8 \text{ kg} \]

Therefore, initial mechanical energy = 7.848 + 1.6 = 9.448 Joules

Using the answer from part b where we calculated the mechanical energy at release, we can calculate the change in energy = 454.78 - 9.448 = 445.334 J

\[ \text{Work} = \text{change in energy} \]
\[ Fd = 445.334 \]
\[ F = 445.334 / 4 \] As \[ d = 4 \text{ m} \]
\[ F = 111.3335 \text{N} \]

The average force applied to the javelin = 111 N

2a. Force acting on cart down ramp due to gravity = \( mg \times \sin 15^\circ \)
\[ = 120 \times 9.81 \times 0.2588 = 304.68 \text{ N} \]

Force acting on cart at right angles to ramp = \( mg \times \cos 15^\circ = 120 \times 9.81 \times 0.9659 = 1137.088 \text{ N} \]
Maximum friction force that could act on cart wheels = $\mu R = 0.01 \times 1137.088 = 11.37$ N

The worker would have to push with $(304.68 - 11.37)$ Newtons just to stop the cart from rolling down the slope. But to stop the cart from rolling down the slope and to actually get it to move up the slope the worker must push with $304.68 + 11.37$ Newtons of force as the friction between the wheels and the ramp will now act in opposition to the movement up the ramp.

**The force that must be applied to the cart must be greater than 316 N**

b. Force acting on worker down ramp = $mg \times \sin 15 = 70 \times 9.81 \times 0.2588 = 177.73$ N  
   Force acting on worker at right angles to ramp = $mg \times \cos 15 = 70 \times 9.81 \times 0.9659 = 663.30$ N  

Maximum friction force acting at shoes of worker = $\mu R = 0.45 \times 663.3 = 298.485$ N  
The force exerted against cart (>$316$ N) will be exerted against worker (Newton’s 3rd) in addition to the 177.73 N acting down the slope due to body weight.

**The force needed to move the cart exceeds the maximum static friction between their shoes and the ramp so yes they would likely slip.**

c. They could direct their push force slightly upwards from the angle of the ramp. This would reduce the effective weight of cart and increase the effective weight of the worker thus reducing the force horizontal to the ramp needed to move the cart and increase the force horizontal to the ramp needed to push the worker backwards (i.e. cause slipping).

3a. If she is experiencing a ground reaction force (GRF) she is in contact with the ground. The graph shows some GRF for a period of 250 milliseconds. This is why this question is only worth one mark.

**She is in contact with the ground for 0.25 seconds**

b. Net impulse is the impulse over and above body weight impulse ($600$ N x 0.25 s). There are two ways to calculate this. One is to calculate the total area under the curve and minus the impulse due to body weight. The other is to zero the graph (the $600$ N becomes the zero line) and calculate the area under this graph. This is what I will do. Note the first and last triangles (shaded fill on the diagram) will be negative impulses.

1\textsuperscript{st} triangle = $\frac{1}{2} \times 0.025 \times -600 = -7.5$  
2\textsuperscript{nd} triangle = $\frac{1}{2} \times 0.025 \times 400 = 5$  
3\textsuperscript{rd} triangle = $\frac{1}{2} \times 0.05 \times 800 = 20$  
4\textsuperscript{th} rectangle = $0.1 \times 800 = 80$  
5\textsuperscript{th} triangle = $\frac{1}{2} \times 0.03 \times 800 = 12$  
6\textsuperscript{th} triangle = $\frac{1}{2} \times 0.02 \times -600 = -6$

**Total net impulse = 103.5 Ns** $(-7.5 + 5 + 20 + 80 + 12 - 6)$
c. Impulse = change in momentum \[F_t = m(v_f - v_i)\]  
But your told \(v_i = -v_i\)  
Therefore, \(F_t = m2v\)  
103.5 = 61.16(2v)  
\(v = 0.846 \text{ m/s}\)

The vertical speed must equal 0.846 m/s

d. If she leaves the ground with the same magnitude of vertical momentum as she enters it (direction obviously reversed), then at the halfway point of the net impulse needed to achieve zero velocity must be zero. \(F_t = m(v_f - v_i)\)  
\(v_i = -0.846 \text{ m/s}\)  
\(F_t = 61.16(0 - (-0.846))\)  
Impulse = 51.75 Ns  
This is half the net impulse for the 0.25 second period. So you would have to determine at what point in time the summing of the areas reaches half the value of the total area calculated above. That is the zero vertical velocity point.  
This would not actually be too difficult to do, although the question didn't ask you to actually calculate this point in time. The impulse for the first 0.1 seconds is 17.5 Ns (-7.5 + 5 + 20 [adding impulses of the first three triangles]). Therefore, we need to find out how much longer it takes to get an additional 34.25 Ns (51.75-17.5).

This will occur somewhere along the large rectangle, which has a constant net force of 800 N. So our equation with the unknown value of time becomes.

\(F_t = 800 \times t = 34.25 \text{ Ns}\).  
\(t = 34.25/800 = 0.0428 \text{ s}\).  
Now add this to the 0.1 seconds that has already elapsed and you get 0.1428 seconds.

So the point of zero vertical velocity occurs at time \(=0.143 \text{ seconds}\)

4 a) When gravity is the only acting external force, a body’s mechanical energy remains constant.

b) A rotating body will maintain a state of constant angular momentum unless acted upon by an external torque (eccentric force, moment)  
Note: You would loose one mark if you wrote “external force”, and/or one mark if you wrote “constant angular velocity”.

c) The rate of change of angular moment of a system is proportional to the external torque acting on the system, and the change takes place in the direction of the applied external torque (moment or couple). Again, do not say force!

5. Third class lever [1]. Favours speed and range of motion [either for 1 mark]

6. a) Impulse (specifically force-impulse but I’d accept just impulse)  
b) Work (I’d accept energy or change in energy)  
c) Work (I’d accept energy or change in energy)

7. a) Joules/second or Watts \(\text{ML}^2\text{T}^{-3}\)  
b) Joules \(\text{ML}^2\text{T}^{-2}\)  
c) Nm \(\text{MLT}^{-2} \times \text{L}\)  
Note: I prefer to differentiate between moment and energy when writing out the fundamental units of these variables. As the L’s are perpendicular to each other \text{MLT}^{-2} \times \text{L} makes more sense than \text{ML}^2\text{T}^{-2}, but both are correct.
8. Conservation of angular momentum is the relevant principle, specifically we are interested in the transfer of angular momentum. The total angular momentum in the horizontal plane must stay constant due to the platform being frictionless. Therefore when he brakes the wheel against his body he will transfer the angular momentum from the wheel to the whole system.

Another way of saying this is that a torque impulse is applied to the wheel to stop it rotating about its longitudinal axis. It therefore exerts an equal and opposite torque impulse on the subject and hence the whole system (the subject, the wheel and the top plate of the turntable) gains the same angular momentum that the wheel previously had. As the moment of inertia of the wheel is less than the moment of inertia of the system, the system will rotate with a lower angular velocity.

9. Conservation of angular momentum [1 mark for mentioning this] is again the principle that must be understood to explain this sequence of events.

In the diagram, the system (the subject, the wheel and the top plate of the turntable) have an angular momentum about a transverse axis (sagittal plane) acting through the axis of the wheel. There is no angular momentum about a longitudinal axis (horizontal plane). As the subject is on a frictionless platform the total angular momentum cannot change and therefore as the wheel is rotated into a horizontal plane he will spin in the same plane with equal and opposite angular momentum. Hence the sum of the angular momentum about a longitudinal axis is still zero. [4]

Another way of explaining it? When he turns the wheel he exerts a torque against the wheel and the wheel obviously exerts an equal and opposite torque against him (Newton’s 3rd law). This torque causes him to spin in the direction described above (in an opposite direction to the way the wheel is spinning)

You could also mention that the original angular momentum is taken up by the entire earth system, as the system above turntable stand cannot rotate about a transverse axis (the turntable is only frictionless in the horizontal plane). It will try to rotate in the sagittal plane by exerting a torque on the stand and earth but the stand will push back to cancel this torque out.

10. a) You are given a muscle moment vs. time graph and asked for an angular velocity. This should have led you to determine this is an angular impulse, angular momentum question.

Note: Remember that if moments and/or forces are given with a time course of action, then it is probably a impulse momentum question. If they are given with linear or angular displacements then it is probably a work-energy question. Of course if they are given with only the masses or moments of inertia with no mention of the time or distance of application then the problem is probably asking for an instantaneous linear of angular acceleration. Part b of this question asks for a peak angular acceleration and this can be calculated using the angular equivalent of \( F = ma \) which is \( \tau = I \alpha \).

Angular impulse (Mt) is the product of moment multiplied by the time the moment is applied. The area under the curve in this question represents Mt. Basically, angular impulse is the integral of the moment with respect to time.
Mt = I(ωf−ωi) (initial angular vel. = zero as we know it starts from a static flexed position)

Angular impulse = Iωf

(0.5)(0.2)(150) = 0.7ωf

ωf = 15/0.7 = 21.429 rad.s⁻¹

Angular velocity of knee at time =0.2 seconds = 21.4 rad.s⁻¹

b) The angular equivalent of F = ma is τ = Iα .

Peak angular acceleration will occur at peak torque.

150 = 0.7α  α = 150/0.7 = 214.29 rad.s⁻²

Peak angular acceleration of knee during the 0.2 seconds = 214 rad.s⁻²

c) The angular equivalent of the power equation Power = force x velocity is:

Power = torque x angular velocity (Power = τω).

The angular velocity of the knee at time = 0.1 seconds can again be calculated using the angular impulse = change in angular moment equation used in part a. However, we can see that the moment-time graph is symmetrical and that the angular velocity at time = 0.1 seconds will be half that of the answer from part a (namely 10.7 rad.s⁻¹)

Therefore the muscle moment power = Mω = (150)(10.7) = 1605 Watts (Joules.s⁻¹)

**Muscle moment power at time 0.1 seconds = 1605 Watts**

11. a) Release velocity = 20 m/s  Mass = 2 kg  Height above ground = 1.5 m

Mechanical energy at release  = ½mv² + mgh = 0.5(2)20² + 2(-9.81)(-1.5)

= 400 + 29.43 = **429.43 Joules**

As we are neglecting air resistance, gravity is the only force acting so the conservation of mechanical energy applies to the discus. Therefore it’s energy just prior to ground impact is also **429 Joules**.

b) Components Horizontal velocity = 20cos40 = 15.321 m/s

Vertical velocity = 20sin40 = 12.856 m/s

\[ t = -12.856 \pm \sqrt{(12.856^2 + (2 \times -9.81 \times -1.5))} \]

The vertical displacement is -1.5 m

\[ t = -12.856 - 13.954 \]

The positive result from the square root is not an option.

\[ t = -26.81 = 2.733 \text{ seconds} \]

**Range = s = V_{horizontal}t  \quad S = 15.321 \times 2.733 = 41.9 \text{ m}**
c) No, this is not a realistic situation. The discus is a highly aerodynamic object and is affected markedly by the air it flies through. Air resistance will obviously slow a cyclist down for example but the affect the air has on the discus is more complex than simply thinking about form drag.

Discuss throwers can in fact throw further into a head wind, which highlights the complexity of the aerodynamics involved. They also throw the discus out with considerable rotation (angular velocity) to help stabilise its flight. So the mechanical energy question in part a) is rather simplistic.

12. This problem is easier than it may first appear. The number of marks allocated to it may give you some idea that it doesn’t require too detailed a solution.

\[
\text{Impulse} = \mathbf{P} = Ft = m(v_f - v_i) \quad \text{and we are told that } v_i = 0.
\]

As we know the mass all we need to calculate is the final velocity. How do we calculate velocity from time-displacement data? Finite differentiation. This really is a question that you have to “see” the solution and then the math becomes very simple. If you cannot see the solution you could be here all day trying to solve it.

Velocity 5 seconds from start \( V_5 = \frac{d_{5.5} - d_{4.5}}{2\Delta t} \)  
\( V_5 = (48.213 - 36.774)/1 = 11.44 \text{ m/s} = \Delta v \)

\[
\therefore \text{Net impulse over the first 5 seconds} = \text{mass} \times 11.44 \text{m/s} = 11.44 \times 70 = 801 \text{ Ns.}
\]

b) Same type of problem. We have the initial velocity which is the velocity at time = 5 seconds and now we must calculate the velocity at time = 10 seconds.

Velocity 10 seconds from start \( V_{10} = \frac{d_{10.5} - d_{9.5}}{2\Delta t} \)
\( V_{10} = (105.514 - 94.111)/1 = 11.40 \text{ m/s} \)

Impulse = \( m(v_f - v_i) \)

\[
\text{So net impulse over the last 5 seconds} = 70 (11.40 - 11.44) = -2.8 \text{ Ns}
\]

This makes sense as the sprinter is at best maintaining his/her velocity over the last few seconds of the race and is not increasing velocity. Therefore you would expect the net impulse to be low and probably to be slightly negative.

c) Going back to our equation and result in part a). \( \text{Impulse} = Ft = 801 \text{ Ns} \)

\[
\text{So the average force over the first five seconds} = F = \frac{801}{5} = 160.2 \text{ N}
\]

We have seen when we looked at sprinting that peak accelerations occur early in the movement. If you remembered this fact it would have reduced how much calculation you would have needed to do.

The peak force is not going to accurately measurable due to the large \( \Delta t \) in the data supplied to you (.5 seconds). However, you have to do the best you can so now we will differentiate over one time frame to increase our sensitivity. Remember that the first velocity you already know, namely \( V_{0.0} = 0 \text{ m/s} \).
Velocity at time = 0.25 s = 
\[ V_{0.25} = d_{0.5} - d_{0.0}/\Delta t \]
\[ V_{10} = (0.857 - 0)/0.5 = 1.715 \text{ m/s} \]

Acceleration at time = 0.125
\[ a_{0.12} = V_{0.25} - V_{0.0}/\Delta t \] (note \( \Delta t \) now equals 0.25 s)
\[ V_{10} = (1.715 - 0)/0.25 = 6.86 \text{ m/s}^2 \]

If you didn’t remember that peak acceleration was likely to occur over the first time frame you could calculate acceleration then you would have to differentiate to get the velocity at time = 1 second. Then differentiate between times 0.25 and 0.75 to get an estimate of the acceleration at time = 0.5 seconds. This will be less than the answer above (5.783 m/s\(^2\) to be exact).

So peak acceleration is know and \( F = ma \) so net peak force = 70 x 6.858 = 480.2 N

Again this is the peak force you can calculate. With more data points you would probably be able to calculate a truer and higher peak force.

**Net peak force over the first 5 seconds = 480 Newtons**

**d)** Probably not as \( \Delta t = 0.5s \). This low sampling frequency (2 hertz) increases the likelihood you would miss the peak force output.

**13. Joint Force Power**
\[ \text{Joint force power} = 2165 \text{ Js}^{-1} \]

The joint force power on the shank would be 500x5xcos150 = -2165 W. This means that the total energy of the shank is reducing and the total energy of the thigh (JFP was positive) is increasing.

**Energy is being transferred from the shank to the femur**

**Power = \Delta energy/time** or **Power = \Delta work/time**

Work can be obtained from the integral of power (power x time)

Therefore: work done = 2165.06 x 0.6 = 1299.04 Joules \([2]\)

**Work done on femur = 1299 J**

**14.** The force-time curve shape is identical to the shape of the acceleration-time curve as the mass of the subject is constant (this is why I restated \( F = ma \) in the question).

Estimate the integral of the acceleration to get velocity. Finite integration to get change in velocity is the area under the curve. So we see a decrease in velocity (integral of the negative side) followed by an increase in velocity which we would estimate gets us back to the same velocity (we’ll assume the subject is not slowing down). The value of the forward velocity (which represents the velocity of the body’s centre of gravity) should not drop too near to zero and obviously never below it so the result opposite makes sense.
15. From a purely mechanical point of view the pulling vector shown in the question is the best option. The force of friction $F_f = \mu R$ so if the normal force to the ground (the effective weight) increases so does the friction. When you push slightly downward there is a vertical component that will end up increasing the frictional resistance. On the other hand the pulling action shown reduces the effective weight of the desk ($R$) as the vertical component of force is upwards.

However, body alignment is also critical and if you round your back to pull you may be putting more stress on some structures (vertebral disks for example) when pulling, compared to pushing with a straight back.

16. If you look at the diver in question 25 you will see one type of activity that this graph could represent. It shows a constant angular momentum ($H = I\omega$) so the net moment acting on the system must be zero. This obviously occurs in airborne activity where air resistance is negligible.

So the graph shows that as the diver tucks the moment of inertia reduces and the angular velocity increases (such that the product of these two variables is constant).

17. a) You are lowering the barbell under control so the deltoids and triceps are the prime movers, working eccentrically.

b) It is obvious when someone lowers a weight slowly that the energy of the system is being reduced. This is because the kinetic energy is relatively constant but the potential energy is reducing. If the energy of the barbell system is being reduced you must be doing negative work on the barbell. Negative work against an external force (in this case gravity) must be being performed by an eccentric contraction.

The barbell system weighs 60 kg. The weight lifter lowers the barbell 0.35 m, at which point it is moving at 0.5 m/s.

c) The barbell was at rest but know it is moving at 0.5 m/s so it has gained 7.5 Joules ($\frac{1}{2}mv^2 = \frac{1}{2} \times 60 \times 0.5^2$) of kinetic energy.

But it is 0.35m lower in the gravitational field so it has lost 206 Joules of potential energy ($mgh = 60 \times -9.81 \times -0.35$). The difference is 206-7.5 = 198.5 Joules.

**Therefore the weightlifter has done 198.5 Joules of work on the barbell**

d) Work is the product of force times distance a scalar quantity. However, if the work is done to reduce (absorb) the mechanical energy of a system we refer to it as negative. (he applied a force over a finite distance on it hence he “did work on it [F x d]). Negative in this sense does not refer to direction.

e) The external mechanical work and metabolic cost of the work are not the same thing.

Even the net mechanical work is the sum of the internal work and the external work and we have clearly not calculated internal work. Remember that internal work is the work done by muscles in moving body segments, overcoming co-contractions and other factors, and external work is the work done by muscles to move external masses or work against external resistance. We have not calculated the work done in lowering the limbs as one example.
Even if we did this we still couldn’t calculate the metabolic cost because human muscle is only about 25% efficient at converting chemical energy into tension. Metabolic efficiency is defined as the ability of a muscle to convert metabolic energy to tension. Mechanical efficiency on the other hand is influenced by C.N.S.’s ability to co-ordinate muscular tension patterns.

Overall muscular efficiency = \( \frac{\text{Net mechanical work}}{\text{Net metabolic energy}} \)

All efficiency calculations involve some measure of mechanical output divided by a measure of metabolic input. **Metabolic work is not too difficult to estimate if we do gas analysis and external work also easy to calculate.** But we need to calculate internal mechanical work. We could calculate the work done in lowering the limbs but some muscles might be contracting isometrically (trunk musculature). How can we calculate the metabolic work they did form mechanics? Co-contractions are also a problem, as we have no way of knowing how much each muscle is working. Also jerky movements with high accelerations and decelerations waste energy compared to gradual acceleration. The generated energy at one joint may be absorbed at another (walking example in lecture). Joint friction must be overcome as well. In short we have not even come close to calculating the total mechanical work done.

18. This question is about a concept relevant to the previous question. The graph of power versus velocity for human muscle is shown on page 19 is you need to refresh your memory (as if you would need to!). However negative velocity is only a convention to graph muscle contraction where positive velocity is a concentric contraction and negative velocity is an eccentric contraction. Remember that power is the rate of change of energy and energy is a scalar so the “negative” in “negative power” does not represent a direction (i.e. power is also a scalar quantity). The negative in this sense means that the total energy of the system is being reduced. For example, if I catch a ball and reduce its kinetic energy to zero, and its potential energy is not increased, then its total mechanical energy is lower than before I did work on it. Note that doing negative work usually involves an eccentric contraction.

19. If I have asked for a briefly answered question there is no need to explain each of these following points in detail. Some possible listed points are obviously all related to the equation \( F = \mu R \) (where \( F \) is the frictional force, \( \mu \) is the coefficient of friction and \( R \) is the normal force):

a) The value of limiting friction (i.e. the nature of the two “dry” surfaces, are the floors slippery what type of footwear do the workers wear, etc?)

b) Do any foreign substances comes between these surfaces (e.g. oil, water, gravel, etc. in other words does the value of limiting friction really mean anything)?

c) Are there are any steep ramps etc. which could affect the friction needed?

d) Do the workers exert horizontal forces (pushing and/or pulling loads) which would also affect the friction required?

e) If you were being very thorough, you could find out the worker’s masses and calculate the average mass so you could determine an average normal force.

20. Components

- Horizontal velocity = 20 \cos 40 = 15.321 \text{ m/s}
- Vertical velocity = 20 \sin 40 = 12.856 \text{ m/s}
Therefore NET impulses = \(\Delta mv\) (impulse-momentum relationship)

As initial velocity = zero, just multiply release velocities by mass (2 kg)

\[
\begin{align*}
\text{Horizontal NET impulse} & = 30.642 \text{ Ns} \\
\text{Vertical NET impulse} & = 25.712 \text{ Ns}
\end{align*}
\]

However, questions asks for the impulse he applies which in the **vertical** must overcome gravitational impulse. So the thrower’s impulse must be greater.

\[\text{Ft + mgt = NET impulse} \quad \text{(where Ft is thrower impulse and mgt is gravitational impulse)}\]

\[\text{Thrower impulse} = \text{Ft} = 25.712 - (2 \times [-9.81] \times 2) = 25.712 + 39.24 = 64.952 \text{ Ns}\]

**Vertical impulse applied by thrower** = 25.712 + 39.24 = 65 Ns

**Horizontal impulse applied by thrower** = 30.6 Ns

21. Distance hip to thigh C of g = 42 \times 0.44 = 18.48 \text{ cm}

Distance knee to thigh C of g = 42 \times 0.56 = 23.52 \text{ cm}

\[x \text{ and } y \text{ components of distance from knee to thigh C of g} = 23.52 \cos 45^\circ = 16.63 \text{ cm}\]

Therefore the location of the centre of gravity of the thigh is (46.63, 65.63)

(Just add 16.63 to both co-ordinates of the knee location)

Distance in y direction from **ankle** to shank C of g location = 44 \times 0.4 = 17.6 \text{ cm}

X location of ankle C of g is still 30 \text{ cm}

Therefore C of g shank = (30, 22.6)  
C of g thigh = (46.63, 65.63)

Moment of the sum = sum of the moments

\[
\begin{align*}
\text{X co-ordinate} \\
M_{\text{total}}gX &= M_{\text{thigh}}g46.63 + M_{\text{shank}}g30 \\
11X &= (7 \times 46.63) + (4 \times 30) = 326.4 + 120 \\
X &= 446.4/11 = 40.58
\end{align*}
\]

This makes sense if you look at the diagram because the thigh is considerably heavier than the shank and foot and will dominate somewhat in determining the whole leg centre of gravity location.

\[
\begin{align*}
\text{Y co-ordinate} \\
M_{\text{total}}gY &= M_{\text{thigh}}g65.63 + M_{\text{shank}}g22.6 \\
11Y &= (7 \times 65.63) + (4 \times 22.6) = 459.4 + 90.4 \\
X &= 549.8/11 = 49.98
\end{align*}
\]

**Co-ordinates of the whole leg centre of gravity in centimetres are** (40.6, 50)

22. a) \(\tau = I\alpha\) and \(I = mk^2\)  
\(\text{I} = 4 \times (0.32^2) = 0.4096 = 0.41\)

Therefore: \(\tau = mk^2\alpha = 0.41 \times 100 = 41 \text{ Nm}\)
b) Net angular impulse = \( \tau t = \Delta I \omega = I(\omega_f - \omega_i) = 0.41(10-0) = 4.1 \text{Nms} \)
Net muscle moment \((t) = \text{net angular impulse}/t = 4.1/0.21 = 19.5 \text{Nm} \)

c) No because gravity would be opposing the motion and his quadriceps would have to overcome this angular impulse. There also may be some co-contraction of the hamstrings, as they would probably help the gluteals stabilise the hip. The quads would have to exert more torque to counteract this.

23. Refer to question 6 in the kinematics section if you are still unsure about part a)

a) Shank angle at frame 6 = 43.6°
Shank angle at frame 8 = 38.1°
Shank angular velocity at frame 7 = -163 degrees/second = 2.85 radians/second
This makes sense as the shank would be rotating clockwise as frame 7 is getting close to toe-off

b) We need to find the height of the centre of gravity of the segment. We have the joint y-values but not the y-value of the centre of gravity.
Distance from distal joint (ankle) to shank centre of gravity = 0.396 x 0.56 = 0.222m
Centre of gravity is 44% of segment length from proximal joint so it is 56% form distal joint
If we find the shank angle at frame 7 (40.7°) we can draw the situation.
\[ y = 0.222 \sin 40.7 = 0.145 \]
No add this to the y-value of the ankle 0.234 and we get: 0.145 + 0.234 = 0.379m
So potential energy of shank at frame 7 = \( mgh = 3 \times (-9.81) \times (-0.379) = 11.2 \) Joules

c) Rotational energy = \( \frac{1}{2}I \omega^2 \)
\[ I = mk^2 = 3(0.21^2) = 0.1323 \text{kgm}^2 \]
We know the angular velocity form part a) but be careful to use radians/second NOT degrees/second.

**Rotational energy = \( \frac{1}{2}I \omega^2 = 0.5 \times 0.1323 \times (2.85)^2 = 0.537 \) Joules**

d) The linear velocity of the centre of gravity of the shank is the vector sum of two components. The linear velocity of the knee with respect to a fixed frame of reference and the linear velocity of the centre of gravity of the shank with respect to the knee. This later value can be calculated using \( v_t = \omega r \) and as this is tangential velocity its direction is 90 degrees to the alignment of the shank in the clockwise direction (we have this information from the previous parts to this question). The velocity of the knee can be calculated using finite differentiation on the knee joint co-ordinate data. The marker in question here is the tibial condyle marker.
24. This question requires understanding of angular impulse (some texts discuss impulse in terms of force-impulse and torque-impulse) and the relationship between angular work and energy (namely $\tau \theta = \Delta \text{rotational energy}$).

a) Torque (moment) impulse equals change in angular momentum
   \[ Mt = \Delta \omega \] (as $\omega_i = 0$)

\[ \text{Torque-impulse} = 0.25 \times (-20) = -5 \text{ kg.m}^2/\text{s} \]

b) Change in angular momentum takes place over 0.1 seconds so just divide by 0.1.

\[ \text{Average torque} = -5/0.1 = -50 \text{ Nm} \]

c) Work = $\tau \theta$ but we don’t yet know what range of angles the movement has taken place over. But we know $\tau = I \alpha$ and we know the average torque.

If we take the average torque value we can calculate an average angular acceleration.

\[-50 = 0.25 \alpha \quad \alpha = -200 \text{ rads/s}^2 \]

You could use either of these two equations ($\theta = \omega_i t + \frac{1}{2} \alpha t^2$ or $\omega_f^2 = \omega_i^2 + 2 \alpha \theta$)

\[-20^2 = 0 + 2(-200)\theta \quad \theta = 400/-400 = -1 \text{ rads} \quad (57.3^\circ \text{ is a reasonable range of movement}) \]

\[ \text{Therefore work done} = \tau \theta = -50 \times -1 = 50 \text{ Joules} \]

Obviously this is a simplification, as the triceps would have to do less work than this, as gravity would contribute a significant amount of angular torque during this movement.

25. This requires a combination of linear and angular kinematics and angular kinetics.

Need to calculate time in air.

Vertical velocity at take $v_{TO} = 5\sin60 = 4.33 \text{ m/s}$ (we know $d = -5$ m and $g = -9.81 \text{ m/s}^2$)

Quadratic

\[ t = -4.33 \pm \sqrt{(4.33^2 + (2 \times -9.81 \times -5))} \quad \text{The vertical displacement is -5 m} \]

\[ t = -4.33 \pm \sqrt{(18.75 + 98.1)} \quad t = -12.856 \pm \sqrt{116.8} \]

\[ t = -4.33 - 10.8 \quad \text{The positive result from the square root is not an option.} \]

\[ t = -15.149.81 = 1.54 \text{ seconds} \]

Angular momentum = $H = I \omega$  \[ H = 35 \text{ kgm}^2/\text{s} \quad \text{and minimum } I = 3 \text{ kgm}^2 \]
Therefore: \[ 35 = 3\omega \quad \omega = \frac{35}{3} = 11.667 \text{ rads/second} \]

This is the MAXIMUM angular velocity and in reality she would take time to get into a tuck and then take time to extend out before entry into the water.

So the time taken for one revolution \[ = \frac{2\pi}{11.667} = 0.538 \text{ seconds.} \]

So to do 2.5 somersaults (she must enter the water headfirst) would take 1.345 seconds and 3.5 would take 1.883 seconds. She is only in the air for 1.54 seconds so she could only **make TWO complete revolutions**.

26. a) Work-energy relationship states that the work done on a system will equal the change in energy of the system. Here we would calculate the change in energy (an increase in potential energy, \( mgh = \text{P.E} \)).

**Work** = \( 75 \times 9.81 \times 0.8 = 588.6 \text{ Joules.} \)

b) Power is rate of change of energy so we would need the time taken for the movement.

c) We are able to generate greater force during an eccentric contraction. See the graphs of the force-velocity relationship in human muscle in the muscle section.

27. a) When giving the coefficient of restitution, both bodies involved in the impact must be specified. [1] The value 1.16 is impossible. [1] The maximal value for \( e \) is 1.0, even in an impossible to achieve perfectly elastic collision. COR does not have any units it is a ratio. [1]

b) \[-e = \sqrt{\frac{h_b}{h_d}}\]

Note is \( h_d \) height dropped from and therefore a negative value (\( h_b \) is positive) so we can change \(-e\) to \( e\) in the following solution

**Therefore** \[ e = \sqrt{\frac{1.1}{2}} = 0.74 \]

c) The nature (mechanical properties) of the wall and ball. [1]

The temperature of the wall and ball. [1]

The velocity of impact may affect the COR as the squash ball is not of uniform rubber (it has a hollow air-filled centre). [1]

28. The definitions do not have to be exactly worded as below but obviously must convey the same exact mechanical meaning.

a) Newton's first law of motion

Every body continues in its state of rest or motion in a straight line unless compelled to change that state by external forces exerted upon it.

b) Work (what units is this measured in?)

The work done on a body by a force is equal to the product of its magnitude and the distance that the body moves in the direction of the force, while the force is being applied to it. Hay Page 97. Joules are the unit of work.
c) Power (explain with reference to human muscular contraction)
Power is the rate of change of energy. It can also be calculated by multiplying force by velocity. In human muscular terms the power output of a muscle is best thought of as the force of contraction multiplied by the speed of contraction.

d) Centripetal Force.
A force directed towards the axis of rotation point (center of rotation)

e) Free Body Diagram
A diagram of a system showing all the forces acting on the system. Only the system of interest should be drawn hence no pairs of forces (action and reaction Newton’s 3rd law) should appear on the diagram.

f) Resistive Force
An external force that acts against (resists) the movement of a system.

g) Static limiting friction (what units is it measured in?)
Friction is a force that arises to oppose a body sliding or tendency to slide across the surface of another body. Before movement starts friction opposes the force tending to move a body. This force increases until a point where the force can no longer be "matched" by friction. This Upper limit in magnitude of the friction is call static limiting friction (or just limiting friction). It is unitless.

h) Pressure (what units is it measured in?)
Force divided by area  Pascals, KiloPascals, N/m², N/cm² etc.

28. Obviously this type of protective equipment comes into play during a collision. The basic concept is that the ball or opponent is moving and has to be brought to a zero (or at least reduced) velocity during the collision. this is going to take force.

One of two equations could have been used to describe how this force is REDUCED by the use of protective equipment.

The first is Work = Fd. The work-energy relationship tells us that work must be done to reduce the Kinetic Energy \( \frac{1}{2}mv^2 \). If I can apply a force over a greater distance I need less force to achieve the same amount of work. Therefore the padding of the protective equipment deforms and reduces the actual force of impact on the player.

This could also be explained using the equation:
Ft (mechanical impulse) = m\(v_f\) - m\(v_i\) (change in momentum)

|Obviously the player or ball has a certain amount of momentum at impact and this can be reduced by applying a high force over a short time or a lower force over a longer time. padding helps achieve the late r of these two options.

The second benefit revolves around the equation Pressure = force/area. When a soft padded area deforms the force of impact is distributed to a slightly larger area (in the case of rigid shin pads the force is distributed to quite a large surface area). Therefore, even given a large force the actual pressure experienced by the players tissues is reduced. As with examples of spiked heels (and knifes) it is not actually the force we have worry about but the pressure.
29. A rotating body will continue to turn about its axis of rotation with constant angular momentum, unless an external couple or eccentric force (torque) is exerted upon it.

To initiate rotation in the air the athlete (cat?) first has to form two axes of rotation as shown. Flexing at the hip can easily achieve this.

The athlete then rotates his/her upper body about a longitudinal axis through the upper body (axis one in the diagram). The moment of inertia of the upper body about this axis is very low.

Note: As $I = \sum mr^2$ (or $mk^2$), the moment of inertia is dependent on both the mass and the distribution of that mass about the axis of rotation. As can be seen from the diagram the mass of the upper body is distributed very close to Axis 1 (i.e. $k1$ is a low value).

Newton’s Third Law (rotational analogue) tells us:

For every torque that is exerted by one body on another there is an equal and opposite torque exerted by the second body on the first.

Therefore the lower body will react to this upper body rotation by rotating in the opposite direction about axis 1. However, the moment of inertia of the lower body about axis 1 is much greater than the moment of inertia of the upper body about axis 1 (i.e. $k2$ is a higher value than $k1$). Thus the torque required to rotate the upper body through say $90^\circ$ will only produce a reaction in the lower body of say $10^\circ$.

This is because the rate of change of angular momentum of a body is proportional to the torque causing it (part of Newton’s 2nd)

$$\tau = I(\omega_f - \omega_i)/t \quad \tau t = I(\omega_f - \omega_i)$$

So if the torque on both the lower and upper bodies are equal and opposite (which they must be) and the moment of inertia of the lower body is much greater than the angular velocity of the lower body initiated about axis 1 must be less (see above equation). Over any given time period then the angular distance travelled will be less ($\Delta\theta = \omega t$).

This argument can then be applied to the lower body rotating about axis 2. Now the lower body is rotated a large amount (to “catch-up” to the upper body) and the reaction in the upper body about axis 2 is very small. By continuing this process the athlete can complete a rotation in the air while his/her angular momentum remains zero and constant.

Note how precise and mechanically based I want the answer to be (don’t assume the person you are explaining to has a good biomechanical background).

31. Sum of the moments (torques) equals moment (torque) of the sum. See the section of centre of gravity segmentation method for more info.

Torques about the left edge of the beam.

Total weight = 195N Let $d$ equal distance to C of g from left edge.

$$(15N \times .1m) + (100N \times .5m) + (80N \times 1m) = d \times 195N$$
1.5 + 50 + 80 = 131.5 = 195d  Therefore  d = 131.5/195 = 0.67m

32. Work-Energy relationship. [Assume all discussion refers to horizontal plane].

Kinetic energy of ball = 0.5mv^2  = 0.5 x 1.3 x 30^2  = 585 Joules

Therefore to reduce kinetic energy to zero in horizontal direction the work done must equal 585 Joules.

When you are given a time and asked to find an average force you must always think IMPULSE.

Initial momentum of ball = mv = 1.3 x 30 = 39 kg.m/s
Therefore change in momentum = 39 kgm/s  (as balls velocity is reduced to zero).

Impulse = change in momentum.  Therefore: Ft = 39  F0.5 = 39  (t = 0.5secs)

Therefore average force equals 78 N. Note that this would not be peak force.

33. a)  \( t = F \times d = 1500 \times 0.03 \cos 60 = 22.5 \text{Nm} \)

b)  \( \tau = I \alpha \)  \( \tau = 22.5 \text{Nm} \)  \( I = mk^2 = 3 \times 0.3^2 = 0.27 \text{kg.m}^2 \)

\( \tau = I \alpha \) Therefore:  \( 22.5 = 0.27 \alpha \)  \( \alpha = 83.33 \text{ radians/sec}^2 \)

34. The total instantaneous mechanical energy of a segment is given by the equation: \( E = \frac{1}{2}mv^2 + \frac{1}{2}Io^2 + mgh \). Let's deal with each of these separately, after we draw the approximate alignment of this segment at frame 3 (opposite).

Potential Energy = mgh  (so the only unknown is h)

We need to calculate L (segment length), then r′ (the distance from the distal joint to the segment centre of gravity) and then find the vertical component of the centre of gravity in relation to the ground.

Find L with Pythagoras \( L^2 = (92.2-53.2)^2 + (130.9-121.3)^2 = 39^2 + 9.6^2 = 1521 + 92.16 = 1613 \)
Therefore \( L = 40.16 \text{ cm} \)

\( r' = 0.57 \times 40.16 = 22.89 \text{ cm} \) (use 57% as c of g is located 43% from proximal joint)

\( \theta_3 = \tan^{-1}(39/9.6) = 76.17^\circ \) (this makes sense: 1st quadrant and aligned quite upright)

So \( h' = 22.89\sin 76.2 = 22.23 \text{ cm} \)
We needed to get \( r' \) anyway so I calculated this value the long way. However, the percentage of the length from the segment to the centre of gravity to the distal joint (57) can just be used on the vertical component of the segment length that you can get easily from the co-ordinates (39 cm). So 57% of 39 cm = 22.23 cm.
Now add \( h' \) to the y-value of the knee to get \( h = 22.23 + 53.2 = 75.43 \, \text{cm} = 0.7543 \, \text{m} \)

So \( mgh = 7 \times (-9.81) \times (-0.7543) = 51.8 \, \text{Joules} \) \( [3] \) (both \( g \) & \( h \) are negative as we calculate the distance to the ground)

**Rotational Kinetic Energy** = \( \frac{1}{2} I \omega^2 \)

\( I_g = mk^2 = 7 \times (0.1297)^2 = 0.118 \, \text{kgm}^2 \) \( \) (k/L = 0.323, so k = 0.4016 \times 0.323 = 0.1297 m)

To get \( \omega \) at frame 3 we need to get \( \theta \) for frame 2 and 4.

\[ \theta_2 = \tan^{-1}\left[\frac{92.6-53.6}{(128.1-118)}\right] = \tan^{-1}(3.86) = 75.48^\circ = 1.317 \, \text{rads} \]

\[ \theta_4 = \tan^{-1}\left[\frac{91.9-52.8}{(133.7-125)}\right] = \tan^{-1}(4.49) = 77.46^\circ = 1.352 \, \text{rads} \]

\[ \omega_3 = \theta_4 - \theta_2/2\Delta t = 77.46 - 75.48 / 0.033333 = 59.25^\circ /s = 1.034 \, \text{rad/s} \]

**Rotational Kinetic Energy** = \( \frac{1}{2} I \omega^2 = \frac{1}{2}(0.118)1.034^2 = 0.0631 \, \text{Joules} \) \( [3] \)

**Translational Kinetic Energy** = \( \frac{1}{2}mv^2 \)

Remember that it is the velocity of the centre of gravity that we need. There are few ways to attack this problem. Using some geometry and trigonometry you could find the co-ordinate position of the centre of gravity of the segment for frames 2 and 4 and then differentiate to get the velocity (most of you will have done it this way). If \( \Delta t \) is adequate this will give the same result (within 1 or 2%) as the method will use below.

However the more elegant method (one that you would use in a program to differentiate a whole batch of data is shown below).

**Position and velocity at frame 3:**

\[ x_g = x_{knee} + r' \cos \theta_3 \]
\[ y_g = y_{knee} + r' \sin \theta_3 \]
\[ \dot{x}_g = \dot{x}_{knee} - r' \dot{\theta}_3 \sin \theta_3 \]
\[ \dot{y}_g = \dot{y}_{knee} + r' \dot{\theta}_3 \cos \theta_3 \]

The dot (') just means the first derivative, I find it is easier to write the equations that way. Check your calculus if you can't follow that differential (explained below).

The proof of this uses the chain rule in calculus

\[ \frac{d(r \cos \theta)}{dt} = r \left( \frac{d \cos \theta}{d \theta} \frac{d \theta}{dt} \right) \]
\[ \frac{d(r \cos \theta)}{dt} = -r \dot{\theta} \sin \theta \]

We already know: \( \theta_3 = 76.17^\circ (1.33 \, \text{rads}) \); \( r' = 0.229 \, \text{m} \); and \( \dot{\theta}_3 = \omega_3 = 1.034 \, \text{rads} / s \)

So we need to calculate the x and y components of knee velocity at frame 3 so we need to differentiate across frames 2 and 4.
\[ x_{\text{knee}} = \frac{(125 \cdot 118)}{0.03333} = 210.02 \text{ cm/s} = 2.1 \text{ m/s} \]
\[ y_{\text{knee}} = \frac{(52.8 - 53.6)}{0.03333} = -24 \text{ cm/s} = -0.24 \text{ m/s} \]
\[ \dot{x}_g = x_{\text{knee}} - r\dot{\theta}_3 \sin \theta_3 \]
\[ \dot{y}_g = y_{\text{knee}} + r\dot{\theta}_3 \cos \theta_3 \]
\[ \dot{x}_g = 2.1 - (0.229)(1.034) \sin 76.17 \quad \dot{y}_g = -0.24 + (0.229)(1.034) \cos 76.17 \]
\[ \dot{x}_g = 2.1 - 0.23 = 1.87 \text{ m/s} \quad \dot{y}_g = -0.24 + 0.057 = -0.183 \text{ m/s} \]

No need to find the resultant as we are going to square the velocity anyway (energy is a scalar remember).

Translational Kinetic Energy = \( \frac{1}{2}mv^2 \) = 0.5(7)(1.87\(^2\) + [-0.183]\(^2\)) = 12.36 Joules \[4\]

Total mechanical energy = \( \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh \) = 12.36 + 0.063 + 51.8 = 64.2 Joules

35. Instantaneous accelerations (no time or distance over which the force acts is given).

So \( F = ma \) and \( \tau = I\alpha \) (don't forget these are net forces and moments)

**Linear Acceleration** (two forces \( F \) and \( mg \))
\[
F_{y_{\text{net}}} = F \sin 97.41^\circ + mg = 1000(0.9916) - 9.81(70) = 991.6 - 686.7 = 304.9 \text{ N}
\]
\[
F_{x_{\text{net}}} = 1000 \cos 97.41^\circ = -129.0 \text{ N}
\]
Resultant \( F_{\text{net}} = \sqrt{(304.9^2 + [-129]^2)} = 331.1 \text{ N} \quad F = ma \) so \( a = 4.73 \text{ m/s}^2 \)
Direction \( \beta = \tan^{-1}(304.9/-129) = -67.07^\circ + 180 = 112.93^\circ \) (2\(^{nd}\) quadrant)

So the acceleration is 4.73 \text{ m/s}^2 at 113\(^\circ\) to right horizontal \[4\]

**Rotational Acceleration** (\( mg \) does not act as a moment, only \( F \))

So the moment arm \( d = rsin28.65^\circ = 1.2(0.479) = 0.575 \text{ m} \)

Moment of force = \( \tau = Fd = 1000(0.575) = 575 \text{ Nm} \) (clockwise)
\[
\tau = I\alpha = -575 = 12\alpha \quad \text{Therefore} a = -575/12 = -47.9 \text{ rad/s}^2
\]

So the angular acceleration is -47.9 rad/s\(^2\) clockwise \[4\]

I would suggest he is performing an inward forward somersault dive off a diving board (possibly a dismount from a bean). He is moving backwards slightly and will not land on the same spot as he took-off from so it is unlikely he is doing a simple forward somersault. However, in the dive he needs to move backwards to avoid hitting the board on the way down! I would be open to other suggestions in such questions, but they must be realistic given the vectors \( a \) and \( \alpha \). \[2\]
36. Centrifugal force = $F_{cf}$
   
   $F_{cf} = \frac{mv^2}{r} = \frac{(80 \times 20^2)}{35} = 914.286$ N

**Force up the slope due to $F_{cf} = F_{cf}\cos20 = 859.15$ N**

However, this is not the only force acting on the system as shown opposite. A component of gravity acts down the slope ($mg\sin20$) and this will mean that not all of the force calculated need be provided by friction.

Force down the slope = $mg\sin20 = 268.42$ N

So **NET force up the slope is only 859.15 - 268.42 = 590.73 N** [2]

If we look at the other components to the two forces in the diagram we can see they both act in the same direction. So the net "downward" force (actually applied at -70° to the horizontal) becomes:

$F_{cf}\sin20 + mg\cos20 = 312.70 + 737.47 = 1050.17$ N

So this is greater than $mg$ [2]

**Explanation [1]:** Well you never get something for nothing in mechanics. The benefits of the banking are that less lean (relative to the alignment of the bank) and less friction are required but you end up with a greater normal force. This is a pretty good trade off as if you have ever been around a tight bend in a roller coaster you realise that you can withstand some pretty high forces when being driven into your seat! On the bike the forces will not be anywhere near as high and you can withstand this "extra weight" easily. On the other hand, if you exceed the limiting friction you may not be able to withstand the ensuing fall as easily.