



Analysis of Internet topology data

Johnson Chen

hchenj@cs.sfu.ca

Communication Network Laboratory
Simon Fraser University

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Road map

- Introduction
- Background
- Routing policies analysis
- Spectral analysis
- Conclusions and contributions
- References

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Introduction

- Data sources:
 - **Route Views**: in North America
 - **RIPE** (Réseaux IP Européens): in Europe
- Goals:
 - geography-related routing policies in Internet
 - cluster characteristics of two datasets
- Approaches:
 - define “**reverse pair**” and use it to study data combined from two datasets
 - spectral analysis

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Internet topology

- Definition
- Topology types:
 - physical topology: physical layout of nodes and links.
 - logical topology: paths the traffic flows from node to node.
- Physical connection \neq logical connection
 - routing policies



Autonomous System (AS)

- Internet is a network of ASs.
- AS:
 - a group of IP prefixes that share the same routing policy
 - Autonomous System Number (ASN)
- BGP (Border Gateway Protocol): inter-AS protocol
 - primary BGP function: exchange network reachability information among BGP systems
 - **routing tables** in routers store reachability information



Inter-AS routing policies

- Goals:
 - balance the traffic on links with other ASs
 - reduce the cost of carrying traffic on these links
- One of the important contributing factors:
 - types of business handled by the AS
 - outgoing traffic: content-provider
 - incoming traffic: access-provider, transit AS

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Internet topology data

- Studies of Internet topology rely on data from:
 - Internet topology generators
 - routing tables
- Internet topology generators
 - random graph
 - structural graph
 - degree-based graph

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Internet topology generators

- Random:
 - Waxman model [*Waxman, 1988*]
- Structural:
 - GT-ITM [*Calvert, 1997*]
 - Tiers model [*Doar, 1996*]
- Degree-based:
 - BA model [*Barabási and Albert, 1999*]
 - BRITE [*Medina, 2001*]
 - BT model [*Bu and Towsley, 2002*]
 - PLRG model [*Aiello, 2000*]
 - Inet2 [*Jin, 2000*] (uses genuine BGP routing tables as default input)

Route Views and RIPE

- Route Views: <http://www.routeviews.org>
 - University of Oregon
 - a script in a central AS collects full BGP routing tables from participating ASs and backbones (mostly in North America)
- RIPE: <http://www.ripe.net/ris>
 - Amsterdam
 - uses several Remote Route Collectors (RRCs) to collect routing information from participating ASs (mostly in Europe)

Laplacian matrix of graphs

- The **Laplacian matrix** of a graph G is defined as:

$$L(G) = D(G) - A(G):$$

$$L(G) = \begin{cases} d_i & \text{if } i = j \\ -1 & \text{if } i \text{ and } j \text{ are adjacent} \\ 0 & \text{otherwise .} \end{cases}$$

- $D(G)$: diagonal matrix of G
- $A(G)$: adjacency matrix of G
- d_i : the degree of node i
- **Eigenvalues** of a matrix M are defined as numbers λ satisfying $Mx = \lambda x$ for a non-zero vector x
- Vector x is called an **eigenvector** of the matrix M belonging to eigenvalue λ

Laplacian matrix of graphs

- The normalized Laplacian matrix $N(G)$ [Chung, 1997]:

$$N(i, j) = \begin{cases} 1 & \text{if } i = j \text{ and } d_i \neq 0 \\ -\frac{1}{\sqrt{d_i d_j}} & \text{if } i \text{ and } j \text{ are adjacent} \\ 0 & \text{otherwise.} \end{cases}$$

Spectral analysis of graphs

- The **second smallest** eigenvalue $a(G)$ of L is called the *algebraic connectivity* of a graph G [*Fiedler, 1973*]
- The **largest** eigenvalue of N is closely connected to clusters in G [*Chung, 1997*]
- “Characteristic valuation” of G : assign elements of the eigenvector to vertices of G as their values [*Fiedler, 1975*]
- Characteristic valuation applications:
 - eigenvector of the **second smallest** eigenvalue in partition problems in graph theory [*Fiedler, 1975*]
 - several **largest eigenvectors** to find AS clusters in Internet [*Mihail, 2003*]

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Internet topology datasets

- Prior research

	Route Views	RIPE
Faloutsos, 1999	Yes	No
Chang, 1999	Yes	Yes
Vukadinović, 2001	Yes	No
Mihail, 2003	Yes	Yes

- We used **Route Views** and **RIPE** datasets collected on one day in May, 2003



Data prepossessing

- Concatenate data files and clear noise from data
 - noise: strange and meaningless symbols
- Extract each line which represents an AS route from data files
 - eliminate route duplicates
 - consider route directions



Statistics for Route Views and RIPE

Number of:	Route Views	RIPE
AS paths	6,398,912	6,375,028
Probed ASs	15,418	15,433
AS pairs	34,878	35,225

- An *AS pair* is a pair of connected ASs
- 99.7 % probed ASs (15,369 of them) in both sets are the same
- 85% of AS pairs in **Route Views** and 84% of AS pairs in **RIPE** (29,477 AS pairs) are the same



Core ASs

- Core ASs: ASs with the largest number of degrees
- Degree of a node is defined as the number of edges incident to a node
- 20 ASs with the largest node degrees
- 80% of the core ASs are identical in **Route Views** and **RIPE**

Rank	Route Views		RIPE	
	AS	Degree	AS	Degree
1	701	2595	701	2448
2	1239	2569	1239	1784
3	7018	1999	7018	1638
4	3561	1036	209	861
5	1	999	3561	705
6	209	863	3356	673
7	3356	662	3549	612
8	3549	617	702	580
9	702	562	2914	561
10	2914	556	1	489
11	6461	498	4589	482
12	4513	468	6461	476
13	4323	315	8220	450
14	16631	294	3303	429
15	6347	291	13237	412
16	8220	289	6730	313
17	3257	277	4323	305
18	4766	263	3257	305
19	3786	263	16631	296
20	7132	258	6347	281



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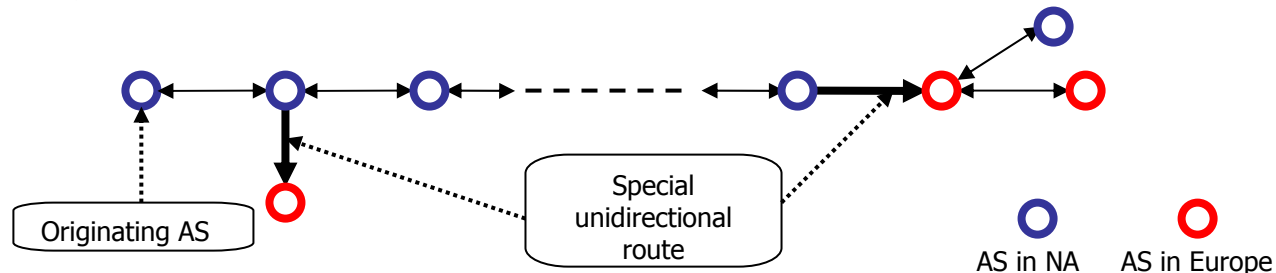


Unidirectional routes

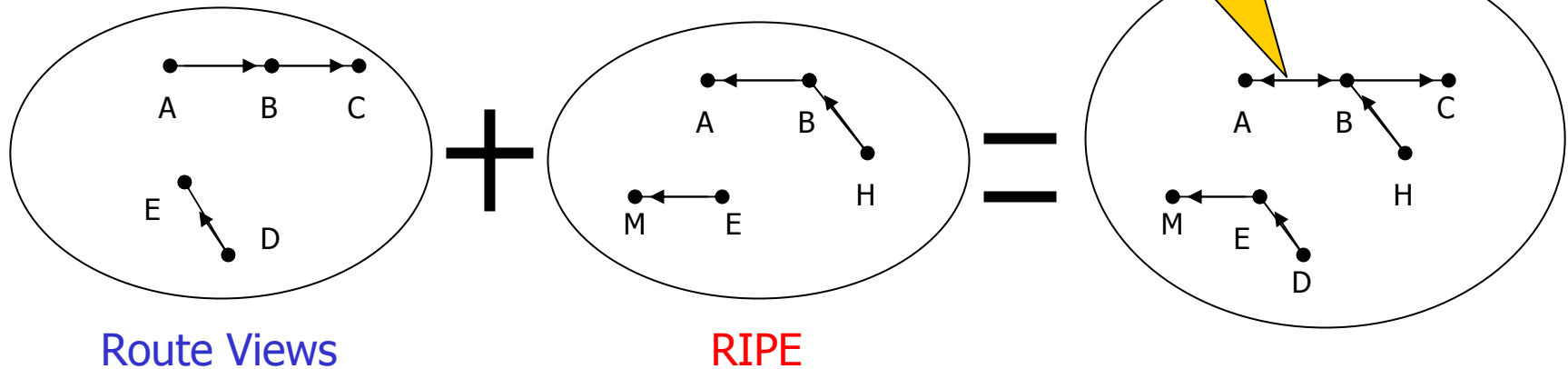
- Most ASs in both datasets are **access-providers**, hence they may prefer that **incoming** traffic be localized to their specific geographic areas
- AS routing policies on incoming traffic influence AS connectivity:
 - we expect to find **unidirectional routes**
 - locating these unidirectional routes may indicate AS routing policies

Special unidirectional routes

- Participating (originating) ASs in **Route Views**:
 - ASs are **access providers** and located in North America
 - ASs prefer **incoming** traffic from ASs in North America over traffic from Europe ASs
 - select ASs in North America as their **next hops** in routing tables
- Same selection process very likely is performed on the chosen ASs until no more ASs in North America can be found, then ASs in Europe are selected
- **Special unidirectional routes** from North America to Europe exist
 - can suggest geography-related routing policies on **incoming** traffic



Reverse pairs



Definition:

Two ASs, A and B , are called a **reverse pair** in two data sets S and T if they satisfy:

- $(A-B) \in (\text{AS pairs in } S)$ and
- $(A-B) \notin (\text{AS pairs in } T)$ and
- $(B-A) \in (\text{AS pairs in } T)$ and
- $(B-A) \notin (\text{AS pairs in } S)$

Properties of reverse pairs

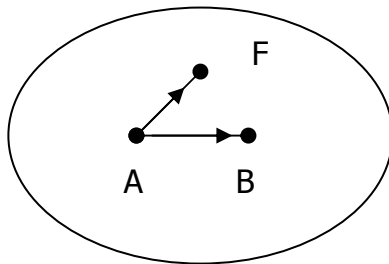
- Reverse pairs are a **subset** of unidirectional routes
- For a reverse pair (AS1, AS2), *outdegree* of AS1 in Route Views is the *indegree* of AS1 in RIPE
 - *Indegree* and *outdegree* are calculated among connections between ASs belonging to reverse pairs
- Hypothesis: reverse pairs can indicate the **special unidirectional routes**
- To test the hypothesis: show that
 - reverse pairs in dataset of **Route Views** have more ASs originating from North America while
 - reverse pairs in dataset of **RIPE** have more ASs originating from Europe

Reverse pair observations

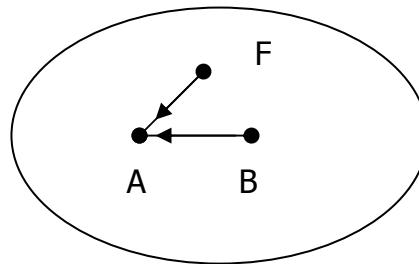
- 558 **reverse pairs** in combined datasets
- **Reverse pairs** represent:
 - 1.60 % AS pairs in **Route Views**
 - 1.58 % AS pairs in **RIPE**
- The number of **reverse pairs** is not negligible
 - both datasets have $\sim 85\%$ of AS pairs in common
 - the proportion of reverse pairs in the remaining 15% distinct AS pairs is not small
- 189 ASs in the **reverse pairs**
- *Degrees* of ASs in the **reverse pairs** range from 1 to 38.

Infer originating AS

- Infer originating ASs in two datasets by *outdegrees* of ASs belonging to reverse pairs
 - an AS that is the originating ASs of two reverse pairs will have an *outdegree* two
- Only calculate *degrees* of ASs in one dataset to infer originating ASs in two datasets
 - for a reverse pair (A, B), *outdegrees* of AS A in Route Views is the *indegrees* of AS A in RIPE



Route Views



RIPE

ASs with *degree* (≥ 10)

RIPE

AS	Outdegree	Indegree	Location
3303	35	3	EU
6730	27	3	EU
3320	24	3	EU
4589	21	1	EU
15412	20	1	EU
3300	19	1	EU
4200	18	1	NA
5400	18	3	EU
8220	17	2	EU
13237	16	2	EU
297	15	0	NA
6762	15	3	EU
13129	14	0	EU
2529	13	1	EU
286	12	1	EU
1759	10	1	EU
6467	10	1	EU

Route Views

AS	Outdegree	Indegree	Location
3257	29	1	EU
6461	26	0	NA
4513	24	0	NA
3356	22	0	NA
3561	18	0	NA
12956	17	0	EU
3246	16	0	EU
3549	15	0	NA
4637	15	0	ASIA
1239	14	0	NA
8001	14	0	NA
2516	13	0	ASIA
2497	12	0	NA
2914	12	0	NA
7911	12	0	NA
3333	11	0	EU
702	10	8	NA
1299	10	3	EU
5511	10	0	EU
6453	10	0	NA



Reverse pair observations

- Each AS occurs primarily as the originating AS among reverse pairs in RIPE or primarily not
- Most originating ASs (15 out of 17) belonging to reverse pairs of **RIPE** are located in Europe
- The majority of originating ASs (12 out of 20) in reverse pairs of **Route Views** dataset are in North America
- **Reverse pairs** can be used to indicate geography-based routing policies on incoming traffic in both datasets
- The effect of geography related AS routing policies for controlling incoming traffic is not negligible

Routes built from reverse pairs

- Two reverse pairs are connected if the originating AS in one pair is the ending AS in the other pair
- Few routes have more than **two** hops and never exceed **three** hops

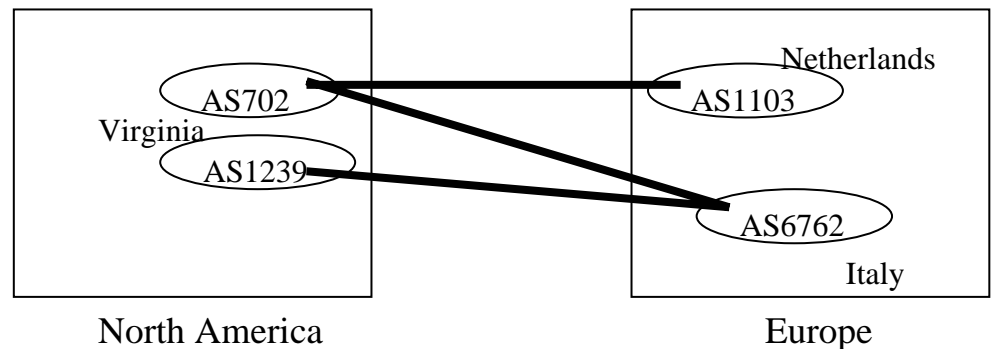
Number of reverse pairs	Number of routes	Number of routes ≥ 2 hops
558	503	56

- 7 out of 503 routes have the maximum number of hops (**three**)
- Internet network operators seem to rarely cooperate when designing routing policies

Routes with more hops

- Routes with more hops are usually geographically dispersed
- Possible reason: **reverse pairs** exist in international links

1103 (EU)	702 (NA)	6762 (EU)	1239 (NA)
8406 (EU)	8210 (EU)	4200 (NA)	3549 (NA)
5417 (EU)	702 (NA)	1299 (EU)	8297 (EU)
6893 (EU)	12541 (EU)	1273 (EU)	4513 (NA)
12381 (EU)	1653 (EU)	2603 (EU)	3257 (EU)
28764 (EU)	24745 (EU)	12713 (EU)	3561 (NA)
15623 (EU)	12755 (EU)	8220 (EU)	3356 (NA)



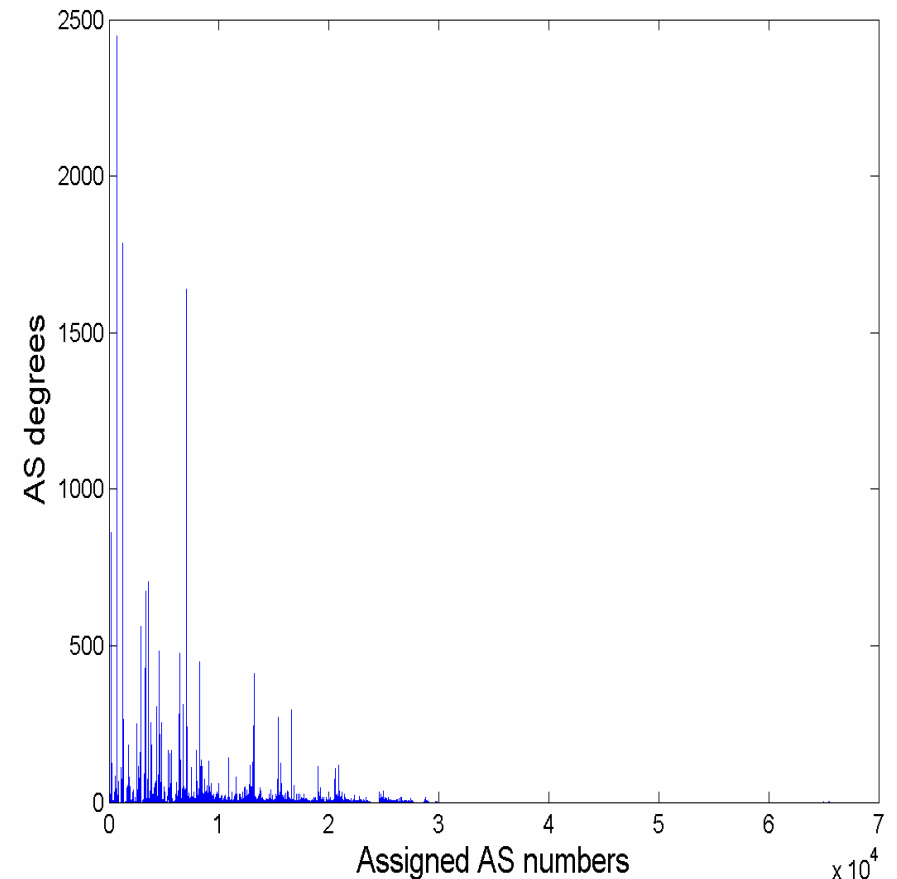
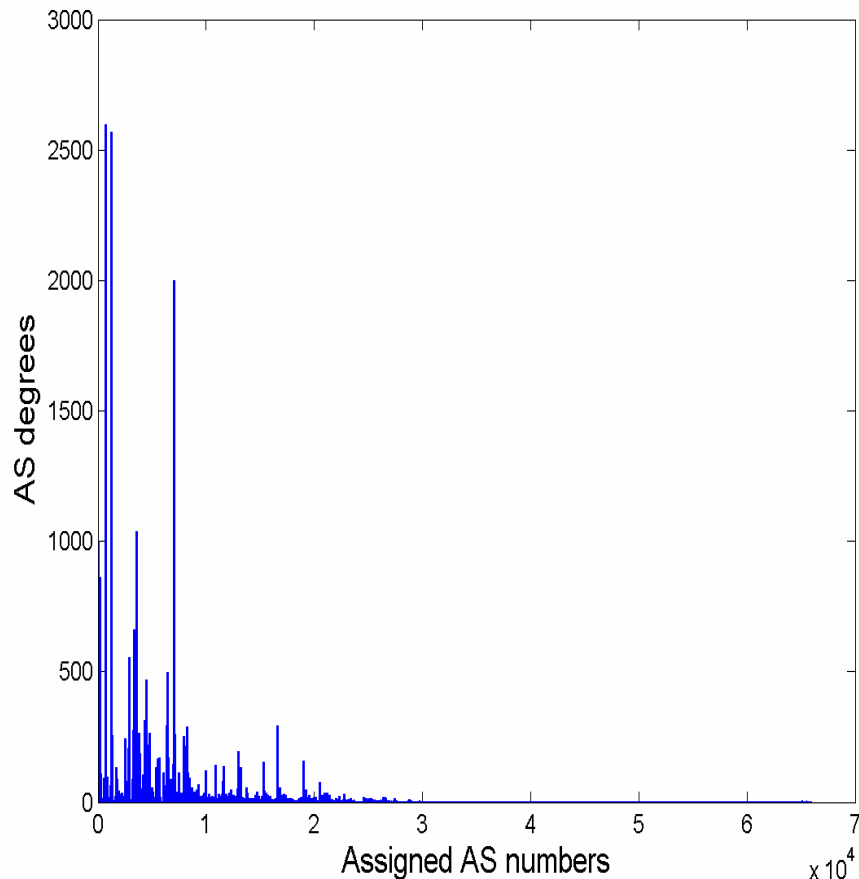
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Spectral analysis of topology data

- Consider ASs with the first 30,000 assigned AS numbers
- AS degree distribution in **Route Views** and **RIPE** datasets:



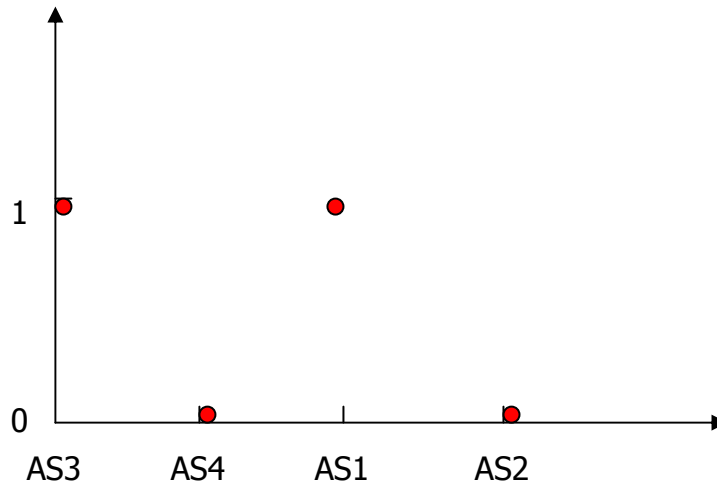
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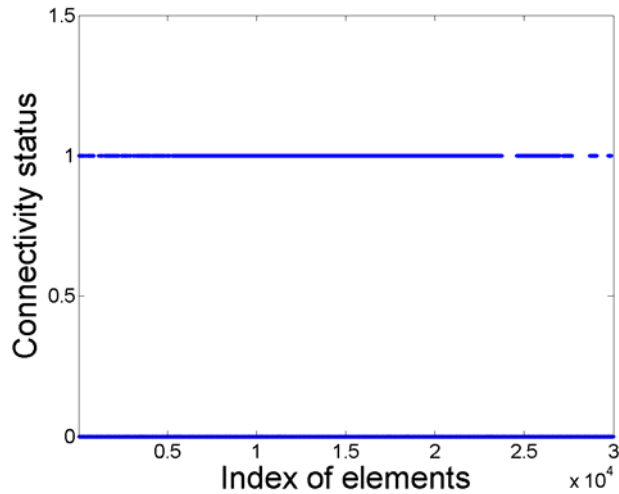
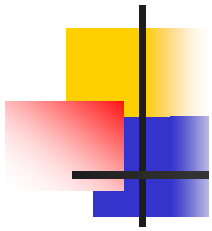
Characteristic valuation

- Build adjacency matrices from preprocessed data
- Calculate the **second smallest** and the **largest** eigenvalues
- Assign elements of an eigenvector to the 30,000 ASs as their values
- Sort ASs in ascending order of elements
- Associate:
 - 1 with connected ASs
 - 0 with isolated or not-found ASs

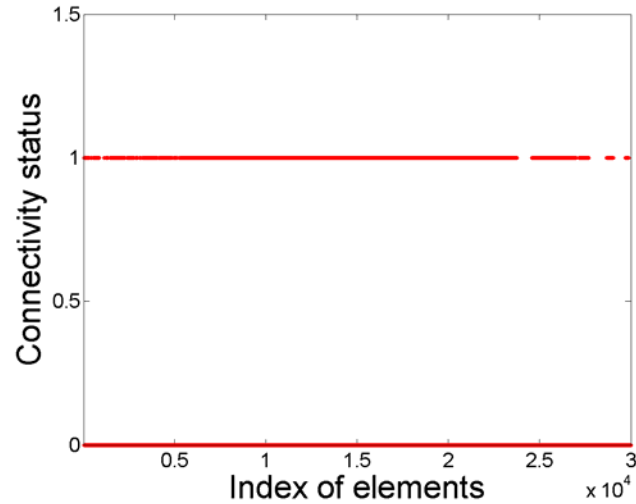
Characteristic valuation

- $[0.1, 0.3, -0.2, 0]$ (the second smallest eigenvector)
- $[AS1(0.1), AS2(0.3), AS3(-0.2), AS4(0)]$
- $[AS3, AS4, AS1, AS2]$ (sort ASs by element value)
- $[\underline{AS3}, AS4, \underline{AS1}, AS2]$



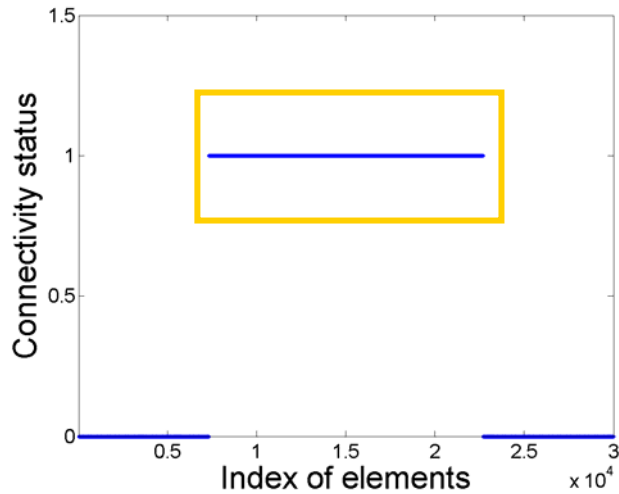


(a) RV_original

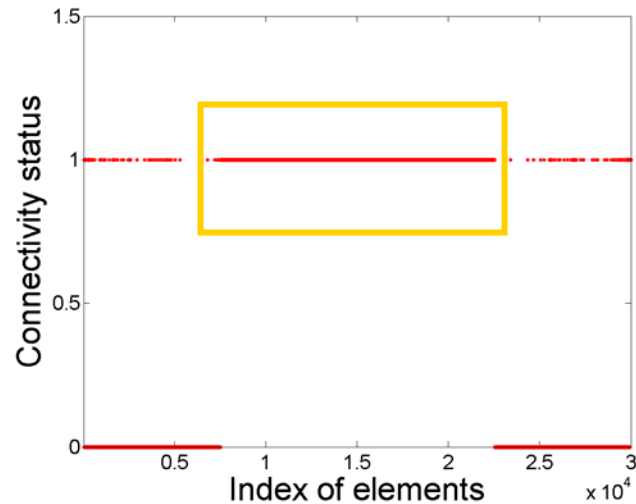


(b) RIPE_original

Before
the sort



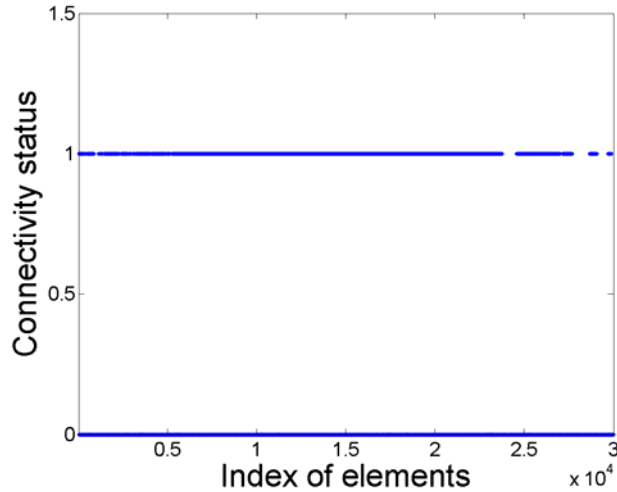
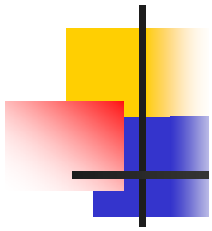
(c) RV_min



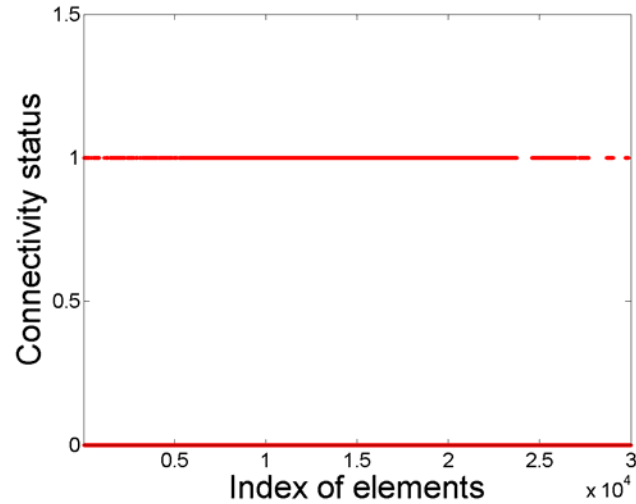
(d) RIPE_min

After
the sort

- Eigenvectors corresponding to the **second smallest** eigenvalues partition connected ASs from disconnected ASs
- In this coarse granularity, two datasets are similar

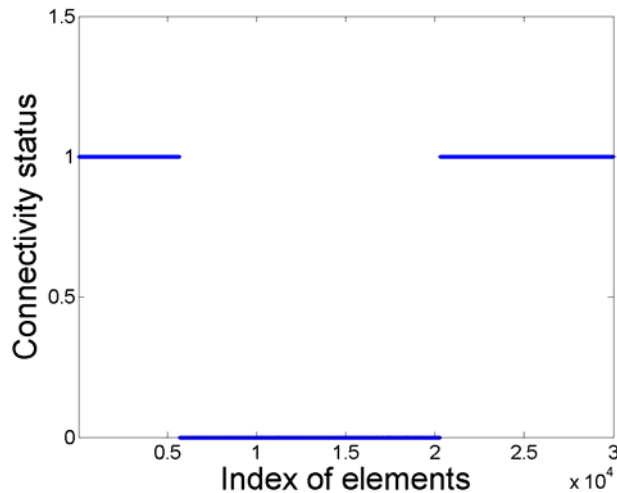


(a) RV_original

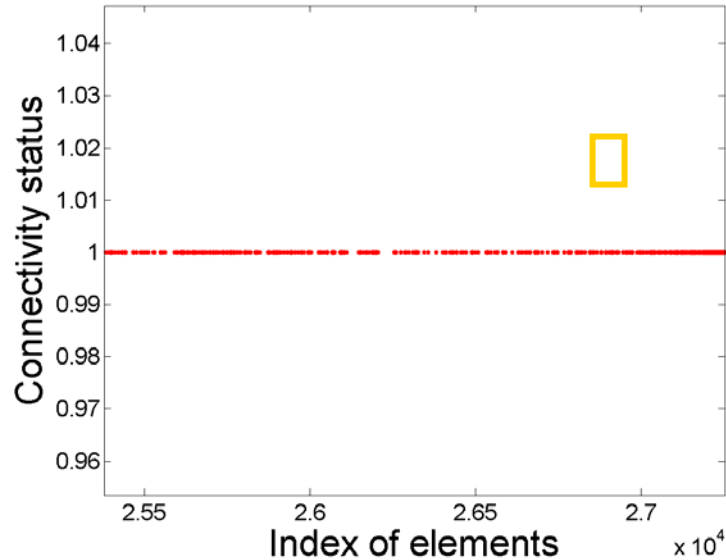


(b) DTDF original

Before the sort



(c) RV_max



- Eigenvectors corresponding to the **largest** eigenvalue can reveal highly connected clusters

- In this finer granularity, two datasets are different



Example of a small cluster in RIPE

Element value	AS	Location
9.87E-02	21032	Germany (TELTA Citynetz Eberswalde)
1.02E-01	2450	France (INRIA-Rocquencourt)
1.02E-01	2426	France (RUBIS Metropolitan Area Network)
1.09E-01	9651	Australia (DOT Communications)
1.09E-01	9652	Australia (ECN Internet)
1.10E-01	16906	Latin America (El Salvador Network)
1.18E-01	13136	Netherlands (Interstroom Informatietechnologie)
1.20E-01	25125	Israel (Israel Local Authorities Data Processing Center)
1.24E-01	21922	USA (Webnet Memphis, Inc.)
1.27E-01	14708	Latin America (WebHost)
1.29E-01	24807	UK (Infocom UK Ltd)
1.61E-01	7566	Australia (Teragen Internet Solutions)
1.77E-01	20908	Poland (CR-MEDIA)
2.01E-01	14647	USA (Network O.S., Inc.)

The same ASs in Route Views dataset

Element value	AS	Element value	AS
-1.06E-05	21032	4.77E-07	9652
-1.38E-05	9651	8.13E-06	13136
-1.61E-05	25125	2.51E-06	20908
-2.11E-05	7566	6.56E-05	14647
-7.58E-05	24807	5.35E-05	2450
		5.35E-05	2426
		3.17E-05	14708
		1.46E-05	16906
		3.1E-04	21922

- They are widely separated into two clusters in Route Views dataset
- The two datasets exhibit different clustering characteristics

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Conclusions and contributions

- We used an unique approach to analyze routing policy practices in Internet by combining two geographically different datasets: **Route Views** and **RIPE**
- We systematically analyzed two sample Internet datasets collected on one day in May 2003:
 - **reverse pairs** were introduced to study geography-based routing policies on incoming traffic
 - we showed that **geographical locations** of ASs may influence Internet routing policies

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Conclusions and contributions

- **Spectral analysis** techniques were employed to study two datasets:
 - **Route Views** and **RIPE** may have distinct clustering characteristics
 - studies regarding clustering should consider dataset being used
- Applications:
 - **geography** should be considered in synthetic Internet topology generators
 - **different clustering characteristics** in two datasets should be considered in topology generators (Inet2) that rely on these genuine topology data

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References: Internet topology and routing

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- Route Views project: <http://www.routeviews.org>.
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