PACKET ERROR PROBABILITY IN TRANSMISSION SCHEME WITH THREE-COPY MAJORITY COMBINING

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Abstract: *In this paper, we derive an analytical expression for the packet error probability in a telecommunication scheme with the three-copy majority combining. The derived expression has a closed form and it is applicable in the case of diversity packet transmission over three independent binary symmetric channels with various bit error probabilities. This expression is then used to analyze an example that illustrates a significant gain of the majority combining compared to the selection combining techniques*.

1. INTRODUCTION

In this paper, we consider a model of telecommunication system shown in Fig. 1. In order to improve the reliability of packet transmission, the space (time or frequency) diversity technique is applied. The transmitter (side A) and the receiver (side B) are connected with three independent channels. Each channel is used for the transmission of an identical copy of the frame containing the user packet and the header with additional bits for error detection, frame numeration, and identification. The transmission is considered successful if at least one of three received copies contains no detectable error or if the implementation of bit-by-bit majority combining procedure yields a frame in which all errors are corrected [1]–[4].

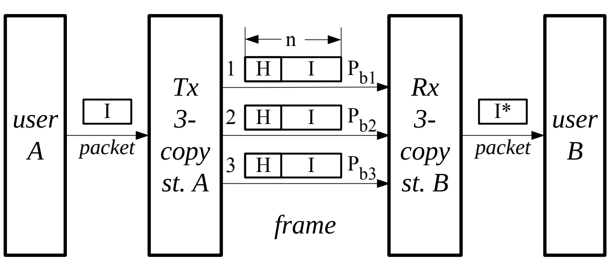


Fig. 1. Proposed model of the three-channel combining scheme.

Our goal is to derive the analytical expression for the probability of packet error transmission. We assume that: (a) *n* is the frame size (with header H and user data I); (b) each independent diversity channel can be modeled as a binary symmetric channel with binary error probability P_{b1}, P_{b2}, and P_{b3}, respectively; (c) the frame synchronization and alignment of the frame at the receiver are perfectly performed, i.e., there are no problems in recognizing the beginning and in identifying each frame.

2. PRELIMINARY NOTATIONS

In this Section, we introduce the following auxiliary specifications and notations useful for the derivation of the packet error probability of three copies majority combining scheme: elementary outcomes, complex outcomes, and collections of complex outcomes.

Elementary outcomes: We first introduce "elementary" outcomes S_i, i = 0, 1, 2, 3, …, 7, that represent possible combinations of errors that can occur by the transmission of bits at the same position of three copies of received frames. These outcomes can be specified by the *j*-th channel error indicators $E(j,i)$, j = 1, 2, 3, as shown in Table I.

TABLE I
SPECIFICATION OF OUTCOMES S_i.

Subscript *i* of the outcome S_i	0	1	2	3	4	5	6	7
Error indicator *E(1,i)* of the first channel	0	1	0	0	1	1	0	1
Error indicator *E(2,i)* of the second channel	0	0	1	0	1	0	1	1
Error indicator *E(3,i)* of the third channel	0	0	0	1	0	1	1	1

Assuming that error indicators $E(j,i)$ have value 1 if there was an error and value 0 in the opposite case, the probability p_i of the outcome S_i is given by:

$$p_i = \prod_{j=1}^{3} P_{bj}^{E(j,i)} \cdot (1 - P_{bj})^{1-E(j,i)}. \quad (1)$$

Complex outcomes: We consider as a possible case of a "complex" outcome $H(k_0, k_1, k_2, k_3, k_4, k_5, k_6, k_7)$, every transmission of three copies of the same frame in which the "elementary" event S_i occurs k_i times. Under assumptions (a) and (b), this "complex" outcome can be considered as a random variable with the multinomial distribution [5] determined by the probability:

$$P(k_0, k_1, k_2, k_3, k_4, k_5, k_6, k_7) = n! \cdot \prod_{i=0}^{7} \frac{p_i^{k_i}}{k_i!}, \quad (2)$$

where $k_0 + k_1 + k_2 + k_3 + k_4 + k_5 + k_6 + k_7 = n$.

Collection of complex outcomes: We specify the collections of "complex" outcomes as those that satisfy specific conditions concerning the numbers k_i of elementary outcomes. Three specific conditions and corresponding notations are introduced: (i) k_i may have any possible value, i.e., $0 \le k_i \le n$ (simply denoted by k_i); (*ii*) k_i may be only positive (denoted by k_i^+); and (*iii*) k_i is equal zero (denoted by 0 instead of k_i). For example, $C(k_0, 0, 0, k_3, 0, k_5^+, 0, 0)$ denotes the probability of collection of "complex" outcomes with constrains: $k_0 \ge 0$, $k_1 = 0$, $k_2 = 0$, $k_3 \ge 0$, $k_4 = 0$, $k_5 > 0$, $k_6 = 0$, and $k_7 = 0$. The probability of this collection we denote as $G(k_0, 0, 0, k_3, 0, k_5^+, 0, 0)$.

3. PACKET ERROR PROBABILITY

Based on the proposed model, the packet error probability P_h of majority combining can be expressed as:

$$P_h = 1 - (Q + R\ ), \tag{3}$$

where Q and R denote the probabilities of two mutually exclusive subsets of events in which the packet transmission is successful.

First subset: The first subset of events consists of all constrained complex events for which, after the transmission of three copies of the same frame, there is at most one erroneously transmitted bit at the same position. The probability of such event can be expressed by:

$$\begin{aligned} Q &= G(k_0, k_1, k_2, k_3, 0, 0, 0, 0) = \\ &= \sum_{k_0}\sum_{k_1}\sum_{k_2}\sum_{k_3} P(k_0, k_1, k_2, k_3, 0, 0, 0, 0), \end{aligned} \tag{4}$$

with the constrains:

$$k_0 + k_1 + k_2 + k_3 = n \text{ and } k_0 \ge 0,\ k_1 \ge 0, k_2 \ge 0,\ k_3 \ge 0 .$$

Substituting (2) in (4) and using the known characteristics of multinomial distribution, we can write:

$$Q = (p_0 + p_1 + p_2 + p_3)^n . \tag{5}$$

Second subset: The second subset of events consists of all constrained complex events for which one copy is transmitted successfully and there are double errors at least on one bit position. Its probability R can be expressed as:

$$R = R_1 + R_2 + R_3 , \tag{6}$$

where:

$$R_1 = G(k_0, 0, k_2, k_3, 0, 0, k_6^+, 0)\ , \tag{7a}$$

$$R_2 = G(k_0, k_1, 0,\ k_3, 0, k_5^+, 0, 0)\ , \tag{7b}$$

$$R_3 = G(k_0, k_1, k_2, 0, k_4^+, 0, 0, 0) . \tag{7c}$$

Since constrained complex events are mutually exclusive, the probability (7a) becomes:

$$\begin{aligned} R_1 &= G(k_0, 0, k_2, k_3, 0, 0, k_6^+, 0) = \\ &= G(k_0, 0, k_2, k_3, 0, 0, k_6, 0) - . \\ &- G(k_0, 0, k_2, k_3, 0, 0, 0, 0) \end{aligned} \tag{8}$$

After substituting (2) in both terms on the right side of (8), we obtain:

$$R_1 = (p_0 + p_2 + p_3 + p_6)^n - (p_0 + p_2 + p_3)^n\ . \tag{9}$$

Similarly, we conclude that the probability (7b) is given by:

$$\begin{aligned} R_2 &= G(k_0, k_1, 0, k_3, 0, k_5, 0, 0) - G(k_0, k_1, 0, k_3, 0, 0, 0, 0) = \\ &= (p_0 + p_1 + p_3 + p_5)^n - (p_0 + p_1 + p_3)^n \end{aligned} . \tag{10}$$

The probability (7c) can be written as:

$$\begin{aligned} R_3 &= G(k_0, k_1, k_2, 0, k_4, 0, 0, 0) - G(k_0, k_1, k_2, 0, 0, 0, 0, 0) = \\ &= (p_0 + p_1 + p_2 + p_4)^n - (p_0 + p_1 + p_2)^n \end{aligned} . \tag{11}$$

Finally, substituting (9)–(11) in (6), and then substituting (5) and (6) in (3), we obtain the analytical expression for the packet error probability:

$$\begin{aligned} P_h = 1 &- (p_0 + p_1 + p_2 + p_3)^n - \\ &- [(p_0 + p_2 + p_3 + p_6)^n - (p_0 + p_2 + p_3)^n] - \\ &- [(p_0 + p_1 + p_3 + p_5)^n - (p_0 + p_1 + p_3)^n] - \\ &- [(p_0 + p_1 + p_2 + p_4)^n - (p_0 + p_1 + p_2)^n] \end{aligned} . \tag{12}$$

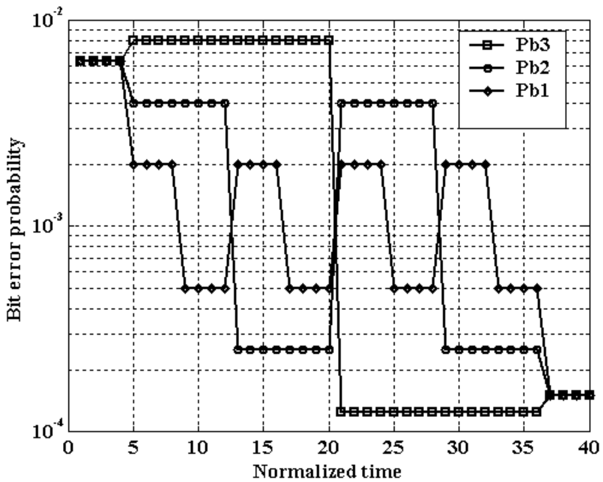
If we assume that the channel characteristics remain constant over the transmission time of the copy of the same packet, the bit error probabilities of logical channels are identical, i.e., $P_{b1} = P_{b2} = P_{b3} = p$. Using (1), we easily find that $p_0 = (1-p)^3$ and $p_1 = p_2 = p_3 = p \cdot (1-p)^2$. Hence, (12) becomes:

$$\begin{aligned} P_h = 1 &- (q^3 + 3pq^2)^n - \\ &- 3 \cdot [(q^3 + 2pq^2 + p^2 q)^n - (q^3 + 2pq^2)^n] \end{aligned} , \tag{13}$$

where $q = 1 - p$.

4. NUMERICAL EXAMPLE

We illustrate the application of the general expression (12) by considering a hypothetical example where the probability errors per bit in the specific channels change over time as shown in the Fig. 2 (top). The time schedule of the individual (specific) changes is chosen so that there are 10 characteristic segments with mutually different probability errors per bit, except in the first and in the last segment where these probabilities have identical values.



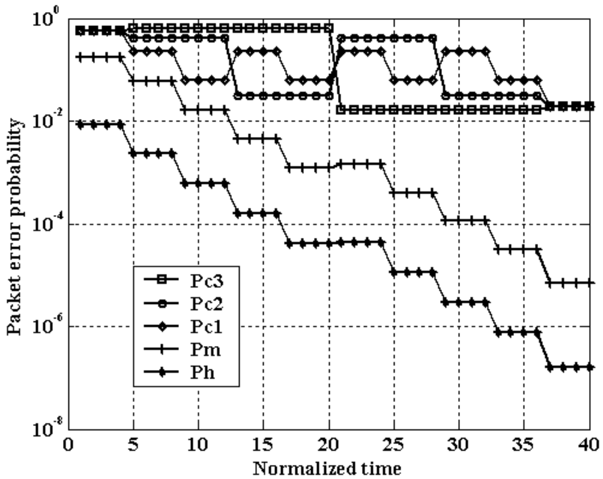


Fig. 2. Example of time sequences illustrating: (top) the bit error probabilities P_{b1}, P_{b2}, and P_{b3}; and (bottom) corresponding packet error probabilities P_{c1}, P_{c2}, and P_{c3} in individual channels and after implementation of the selection P_m and majority P_h combining techniques.

The changes of the corresponding values of packet error probabilities are shown in Fig. 2 (bottom). The three graphs in the upper section marked with P_{c1}, P_{c2}, and P_{c3} ($P_{cj} = [1-(1-P_{bj})^n]$ and n=128), represent packet error probability in the individual channels of the model. Also shown is the packet error probability diagram P_m in the case when the ordinary simple selection combining technique is applied [2]. In this technique, the packet transmission is considered incorrect when the error is detected in each of the three transmitted copies of the same frame. The packet error probability is $P_m = P_{c1} \cdot P_{c2} \cdot P_{c3}$. The last graph shows the packet error probability P_h with majority combining (12).

This example illustrates a significant gain of the majority technique compared to the ordinary diversity. In the example shown in the Fig. 2, this gain (expressed as ratio P_m / P_h) is approximately 15–40. However, the detailed analysis has shown that the gain dependents significantly on both n and P_{b1}, P_{b2}, and P_{b3} of the individual channels.

5. CONCLUDING REMARKS

The derived expression for packet error probability has a closed form. Its numerical evaluation is rather simple, except for extremely large frame size. Consequently, we believe that this expression will find application in the exact numerical analysis of many interesting transmission schemes based on majority combining technique with three independent transmission channels [2]–[4]. Furthermore, the applied analytical approach presented in this paper may serve as a basis for deriving the approximate expressions for estimation of packet error probability in schemes employing other combining techniques [1]–[4].

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