Limitations of Criteria for Testing Transistor Circuits for Multiple DC Operating Points

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***Abstract*— In this paper we address the problem of determining whether a transistor circuit possesses multiple dc operating points. We investigate how the sign change of the determinant corresponding to the Jacobian matrix associated with circuit equations can be used to indicate the number of dc operating points in a transistor circuits. We give circuit examples that illustrate that these criteria may not be reliable, and show that resistor values that determine the number of a circuit's dc operating points, may or may not affect the value of the calculated determinant. Even if the mere existence of a feedback structure depends on whether a particular resistor is open- or short-circuited, the resistor value may not affect the value of the determinant. Hence, the determinant criteria is not always indicative of the number of a circuit's dc operating points.**

I. Introduction

The mathematical problem of finding a nonlinear circuit's dc operating points is described by a set of nonlinear algebraic equations constructed by applying Kirchhoff's voltage and current laws, and by employing the characteristics of the circuits elements. Common numerical approaches for finding these operating points (e.g., Newton-Raphson method) require that the starting point of a numerical process be close enough to the unknown solution. Hence, in order to apply these methods, it is necessary to determine:

- the existence of *multiple* dc operating points
- a good *starting point* for each solution that need to be found.

Both problems are difficult and remain unsolved.

In this paper we address the problem of finding a reliable criteria to determine whether a circuit possesses multiple dc operating points. Although we confine our analysis to bipolar-transistor circuits, our findings are also applicable to other types of transistor circuits.

Purely topological approaches for finding whether a circuit possesses multiple dc operating points, such as detecting the presence of feedback structures [6] that we briefly describe in Section II., are not sufficient. Another approach, the determinant criteria described in Section III., based on the change of sign of the determinant of the Jacobian matrix [14], does not take into account the contribution of sources because their values do not appear in the expression for the determinant [3]. We examine in more details this approach, and in Section IV. we give circuit examples that illustrate that these criteria cannot reliably indicate the number of a circuit's dc operating points.

II. TOPOLOGICAL APPROACH

One approach to qualitatively analyze circuits is to consider only the circuit's *topology*, i.e., only the way circuit's elements are connected together. Nielsen and Willson have discovered [6] a unique topological structure, called feedback structure, that may cause multiple dc operating points in transistor circuits. Such a structure, consisting of bipolar transistors, is shown in Fig. 1.

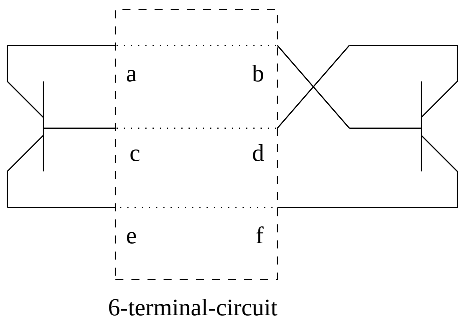

Fig. 1. Feedback structure consisting of two bipolar transistors. The remaining of the circuit is represented as a black box. The type of the transistors (n type or p type) is irrelevant.

Presence of at least one feedback structure is necessary if a transistor circuit is to possess multiple dc operating points [6]. Unfortunately, this is only a necessary condition. Even if a feedback structure is present, the circuit *may* or *may not* possess multiple dc operating points, depending on circuit parameters and biasing conditions [9]–[11].

III. JACOBIAN DETERMINANT CRITERIA

Circuit equations for a transistor circuit consisting of p transistors and q diodes, can be written in the from [15]:

$$\mathbf{QTF}(\mathbf{v}) + \mathbf{Pv} + \mathbf{c} = \mathbf{0}, \tag{1}$$

where

$$\mathbf{F}(\mathbf{v}) := \begin{bmatrix} f_1(v_1) \\ \vdots \\ f_{2p+q}(v_{2p+q}) \end{bmatrix} \tag{2}$$

and $f_i(v_i)$ are monotone increasing functions of the form

$$f_i(v_i) = m_i(e^{n_i v_i} - 1). \tag{3}$$

Matrices $\mathbf{P}$, $\mathbf{Q}$, and $\mathbf{T}$ are constant matrices of order $(2p+q) \times (2p+q)$. $\mathbf{T}$ is a block-diagonal matrix whose first p blocks are 2×2 block matrices of the form

$$\begin{bmatrix} 1 & -\alpha_{r\{2i\}} \\ -\alpha_{f\{2i-1\}} & 1 \end{bmatrix}, \quad 1 \leq i \leq p, \tag{4}$$

followed by q 1×1 blocks, each equal to 1. The controlled-source current-gains satisfy condition $0 \leq \alpha_{f\{2i-1\}}, \alpha_{r\{2i\}} < 1$.

The corresponding circuit may possess multiple dc operating points only if the determinant of the Jacobian matrix can change its sign. We refer to this test as the "determinant criteria." The Jacobian matrix of (1) is

$$\mathbf{J} = \mathbf{QTD} + \mathbf{P}, \tag{5}$$

where $\mathbf{D}$ is a diagonal matrix consisting of the derivatives of the nonlinear function $\mathbf{F}(\mathbf{v})$. In case of a two-transistor circuit, its determinant can be decomposed as [5]:

$$det(\mathbf{QTD} + \mathbf{P}) = c^{1234}d_1d_2d_3d_4 + c^{123}d_1d_2d_3 + c^{124}d_1d_2d_4 + ... + c^{12}d_1d_2 + ... + c^1 d_1 + ... + c^0 d_0, \tag{6}$$

where $d_i \geq 0, i = 1, 2, 3, 4$, if all $f_i(v)$ in $\mathbf{F}(\mathbf{v})$ are monotone increasing functions.

In case of a two-transistor circuit, matrices $\mathbf{P}$ and $\mathbf{Q}$ in (1) can be chosen so that all coefficients c^s, except possibly one coefficient associated with a specific feedback structure, are nonnegative [5]. We will call the "coefficient criteria" the test whether such a coefficient is negative for a specific choice of circuit parameters.

IV. EFFECT OF RESISTORS ON THE VALUE OF THE COEFFICIENTS

A crucial question when applying the determinant and the coefficient criteria is their reliability. Hence, we revisit here these two criteria and we investigate whether the determinant (6) and the sign of the coefficients c^s can be used to reliably indicated changes in the number of a circuit's dc operating points caused by a change of a resistor's value. In other words, we pose the question: Can every change in the number of dc operating points be detected by these two criteria?

In order to evaluate the reliability of the coefficient criteria, we employ the results that the presence of a feedback structure is necessary for the existence of multiple dc operating points [6]. We distinguish two topological cases:

- The presence of a feedback structures does not depend on the resistor's value, i.e., open-circuiting or short-circuiting a resistor does not affect the feedback structure.
- At least one feedback structure gets created or it vanishes if a particular resistor is either open-circuited or short-circuited.

Based on these two cases, we construct circuit examples to illustrate the failure of the coefficient criteria to predict the circuit's behavior.

A. *Example 1: Resistor values do not affect the value of a coefficient used to test the circuit for multiple dc operating points.*

The presence of a negative differential resistance (NDR) that may be observed across one-ports formed from the original circuit, is intimately related to the number of the circuit's dc operating points. The existence of NDR indicates that the circuit may possesses multiple dc operating points for an appropriate choice of the values of independent current or voltage source. Examples how circuits parameters and biasing affect the occurrence of NDR in two-transistor circuits are given in [9]–[11]. In [11], it was shown that resistor values do not necessarily cause more prominent NDR behavior, even if they cause a more prominent feedback structure. Based on examples reported there, we created the example shown in Fig. 2.

For this circuit, matrices defined in (1) are:

$$\mathbf{Q} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \\ 0 & -1 & 1 & 1 \end{pmatrix}$$

$$\mathbf{P} = \begin{pmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ G_2 & G_3 & G_1 & -G_1 \\ 0 & 0 & 0 & G_3 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} V \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

It has been found that for a two-transistor circuit, only one term in the determinant (6) may be negative [4], [5]. For the feedback structure where two emitters are connected to each other, the only possibly negative coefficient

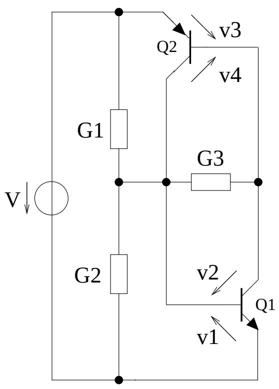

Fig. 2. S-type NDR circuit: if present, the feedback structure causes the circuit to exhibit NDR if values of conductances are appropriately chosen. Any two of the three resistors may be open-circuited. The NDR disappears if all three resistors are open-circuited, even though the feedback structure exists.

is c^{13}. This coefficient can be obtained by calculating the determinant of a matrix formed by taking the 1st and the 3rd column from matrix $\mathbf{QT}$, and all the remaining columns from matrix $\mathbf{P}$:

$$\mathbf{C} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1-\alpha_{f1} & G_3 & \alpha_{f3} & -G_1 \\ \alpha_{f1} & 0 & 1-\alpha_{f3} & G_3 \end{pmatrix}.$$

Then,

$$c^{13} := det(\mathbf{C}) = 1 - \alpha_{f1} - \alpha_{f3} < 0,$$

if $\alpha_{f1} > 0.5$ and $\alpha_{f3} > 0.5$, which holds for realistic transistor parameters. Based on this criteria, we may conclude that the circuit *may* possess multiple dc operating points.

If none of the three resistors is short-circuited, the feedback structure is present. Note, however, that the values of three conductances G_1, G_2, and G_3 do not affect the value of the coefficient c^{13}. Hence, its sign fails to indicate how the values of the resistors affect the number of transistors' dc operating points. For example, it can be easily verified by SPICE simulations [7], [8] that the circuit exhibits:

- NDR behavior [1], [2], [13], and hence possesses multiple dc operating points, for $G_1 = G_2 = 0$ and $G_3 = 0.02$ S, and for $G_1 = G_3 = 0$ and $G_2 = 0.0033$ S.

- no NDR behavior [12], [13], and hence possesses a unique dc operating point, for $G_1 = G_2 = G_3 = 0$.

Nevertheless, the value of the one possibly negative coefficient c^{13} is not affected by the resistor values.

B. *Example 2: Coefficient used to test the circuit for multiple dc operating points does not indicate the disappearance of a feedback structure.*

There are two general ways to destroy a feedback structure. By

- opening of one of the three paths between the transistors, or

- short-circuiting connections between two of the three paths in the feedback structure.

We show now that the value of the only possibly negative coefficient in determinant (6) fails to indicate that a feedback structure *vanishes.*

The circuit example is shown in Fig. 3.

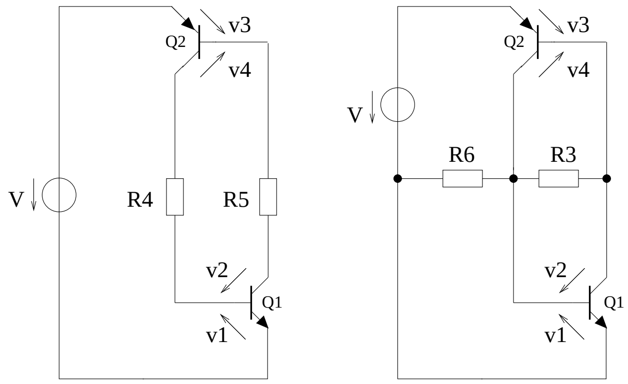

Fig. 3. The coefficient criteria *may fail* to predict the presence of a feedback structure (left), or it *may successfully* do so (right).

Equations for the circuit on the left are defined by matrices:

$$\mathbf{Q} = \begin{pmatrix} 0 & -R_5 & 0 & 0 \\ 0 & R_5 & 0 & R_4 \\ 1 & 1 & 0 & -1 \\ 0 & -1 & 1 & 1 \end{pmatrix}$$

$$\mathbf{P} = \begin{pmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{c} = \begin{pmatrix} V \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

This circuit has an emitter-emitter feedback structure, and the only possibly negative coefficient is c^{13}. It can be calculated by first forming matrix $\mathbf{C}$ by taking the 1st and the 3rd columns of matrix $\mathbf{QT}$, and the remaining columns from $\mathbf{P}$:

$$\mathbf{C} = \begin{pmatrix} \alpha_{f1}R_5 & -1 & 0 & 0 \\ -\alpha_{f1}R_5 & 1 & -\alpha_{f3}R_4 & 1 \\ 1-\alpha_{f1} & 0 & \alpha_{f3} & 0 \\ \alpha_{f1} & 0 & 1-\alpha_{f3} & 0 \end{pmatrix}.$$

Hence,

$$c^{13} := det(\mathbf{C}) = 1 - \alpha_{f1} - \alpha_{f3} < 0$$

if $\alpha_{f1} > 0.5$ and $\alpha_{f3} > 0.5$, which is the case for realistic values of transistor parameters.

Note, however, that although the feedback structure vanishes if one of the resistors (R_4 or R_5) is open-circuited, this determinant, being independent of R_4 and R_5, fails to indicate that.

Unfortunately, even this result can not be generalized, as illustrated by the example of Fig. 3 (right). For this circuit:

$$\mathbf{Q} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & R_3 & -R_3 & -R_3 \\ R_6 & 0 & R_6 & 0 \end{pmatrix}$$

$$\mathbf{P} = \begin{pmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{c} = \begin{pmatrix} V \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Again, matrix $\mathbf{C}$ is formed by taking the 1st and the 3rd columns from $\mathbf{QT}$, and the remaining columns from $\mathbf{P}$:

$$\mathbf{C} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ -\alpha_{f1}R_3 & 1 & -(1-\alpha_{f3})R_3 & 0 \\ R_6 & 0 & R_6 & 0 \end{pmatrix}.$$

Hence,

$$c^{13} := det(\mathbf{C}) = R_3 R_6 (1 - \alpha_{f1} - \alpha_{f3}).$$

This coefficient vanishes if $R_3 = 0$ and/or $R_6 = 0$. Hence, we conclude that appropriate values of R_3 and R_6 may cause multiple dc operating points, as correctly indicated by the sign of the coefficient c^{13}.

V. CONCLUDING REMARKS

In this paper we re-examined the effect of a circuit's resistors on the value of certain determinants and coefficients that can be used to predict whether a circuit may possess multiple dc operating points.

Using example circuits, we showed that changes in the number of dc operating points that may be caused by resistor values, cannot always be indicated by the coefficient criteria. This finding holds no matter whether or not the existence of a feedback structure is affected by the change in the resistor values. Even though the resistors' values affect the entries of matrices $\mathbf{P}$ and $\mathbf{Q}$ in (6), the coefficient criteria do not necessarily indicate a change in the number of dc operating points caused by resistor values.

It has been known that the determinant criteria are not sufficient to reliably indicate the number of a circuit's dc operating points [10], [11]. The reason is that they take into account neither values of dc sources [3], nor the *exact* values of the nonlinearities $f_i(v_i)$ in (2), that ultimately affect the value of the Jacobian determinant (6). Therefore, they ignore the actual values of d_i in (6), which are determined by transistors' biasing, and play an essential role in a circuit's capability to possess multiple dc operating points. Even if it is possible to force the Jacobian determinant (6) to becomes negative, the biasing necessary for this to happen may be impossible for realistic transistor circuits. This biasing information is ignored by the determinant and the coefficient criteria.

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