METHOD PFBS FOR THE ANALYSIS OF TRANSISTOR CIRCUITS: PROOF OF UNIQUENESS

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ABSTRACT: Neither numerical nor topological methods alone can ensure finding *all* dc-operating points of a given circuit of even limited complexity. We have developed a method to help with the numerical analysis of circuits *after* their topology has been analyzed. In this paper, we explain the background of the proposed method and we prove that our method can detect a transistor that belongs to a feedback structure causing multiple dc-operating points.

INTRODUCTION

In this paper, we deal with determining dc operation points of a given circuit. Circuit simulators use approximate methods (e.g., Newton-Raphson algorithm) to determine a circuit's dc operating points. These methods require "good" starting points that are close enough to the unknown solutions. Although in many cases default initial values are sufficient for simulators to converge to a solution, they fail to reliably indicate the existence of multiple solutions. Without a priori knowledge neither all solutions themselves nor a mere convergence area for each solution can be determined.
This problem motivated us to devise a method that attempts to extract useful information about the circuit *before* the computational process begins. One criteria for extracting such information is provided by certain topological results related to transistor circuits. Hence, instead of aiming first at an numerical (quantitative) solution, we first analyze the circuit's structure qualitatively. Then, at a later stage, we refine the approach by performing the proposed numerical analysis method called PFBS method, in order to find the effect of the circuit's structure on the number of dc operating points. (PFBS acronym stands for positive feedback structure.)

PROBLEM STATEMENT

DC operating points are solutions (i.e., zeros) of a set of nonlinear equations [6]

$$\mathbf{F}(\mathbf{x}) = \mathbf{0}$$

constructed from

- Kirchhoff's Voltage Law
$$\mathbf{B}^T\mathbf{u} = \mathbf{0}$$
- Kirchhoff's Current Law
$$\mathbf{A}\mathbf{i} = \mathbf{0}$$
- and the characteristics of the circuits elements
$$f_k(\mathbf{i}, \mathbf{u}) = 0.$$

These equations must be solved during the design process of virtually any electronic circuit.

All current numerical methods for solving nonlinear equations fail to find solutions in *some* cases. This is especially true if there are multiple solutions. Applying standard methods usually does not give *all* solutions. Even the problem of only estimating the number of operating points has not been solved until now [5]. These difficulties imply that solving a set of equations purely numerically means ignoring the circuits electrical properties and peculiarities, which we intend to take into account when applying our method.

TOPOLOGICAL APPROACH

One approach to qualitatively analyze circuits is confining the circuit to the mere *topology* [7], which means considering only the way circuit's elements are connected to each other.

Nielsen and Willson [8] have found a unique structure that may cause multiple dc operating points in transistor circuits called a feedback structure (FS). Feedback structures consisting of bipolar and field effect transistors are shown in Fig. 1.

At least one feedback structure is necessary ([8], [10]) for a transistor circuit to possess multiple operating points. Unfortunately, this is not a sufficient condition. If a feedback structure is present, the circuit *may* or *may not* have multiple operating points, depending on circuit parameters and biasing conditions.

Topological investigations may in some cases be sufficient to determine the number of operating points.

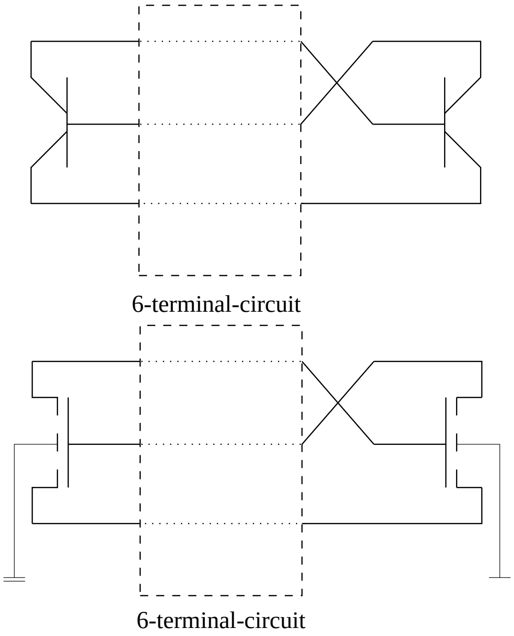

Figure 1: Feedback structure consisting of two bipolar transistors (top), and two FET's (bottom). The rest of the circuit is enclosed in a box. The type of a transistor (n type or p type, n-channel or p-channel) is irrelevant.

In general, however, they fail: either because the topological criteria are not sufficient, or because they ignore numerical information contributing to a circuit's behavior. Some results ([9]) indicate that multiple operating points are only possible if the circuit has *positive* feedback, i.e., circuits with only negative feedback possesses a unique operating point.

THE METHOD PFBS

The type of feedback structure can be analyzed by cutting open the feedback structure and by replacing one of the feedback transistors by a test circuit, as shown in Fig. 2 for bipolar transistors, and in Fig. 3 for FET's. In [1], [3], and [4] we showed that to locate an operating point, it is sufficient to detect that $v_{out} = v_{in}$.

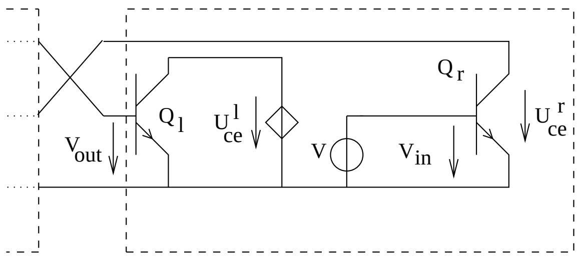

Figure 2: Replacing the bipolar FS-transistor (npn type) by a test circuit.

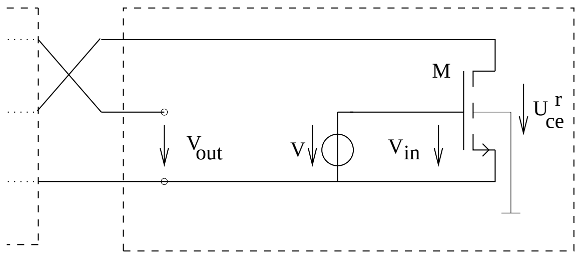

Figure 3: Simplified circuit for testing an n type MOSFET FS-transistor.

Mechanism of the method

We first observe the behavior of the circuit if voltages $V = v_{be} = v_{in}$ increase. Note that transistor Q_r can be biased at any base-current $i_b(Q_r)$ supplied by the ideal voltage source and being dependent on the sources value, with $i_b(Q_r)$ controlling the collector-emitter-current. Hence, for transistor Q_r, the conditions at an operating point are fulfilled for any value of v_{in}.

Let us consider the situation when voltage V coincides *with an operating point*, i.e., $V = v_{in} = v_{be,OP}$. This implies that transistor Q_r (on the right) is biased exactly as the original FS-transistor *before* the replacement. That is true because $v_{in} = v_{be,OP}$ and $i_b(Q_r) = i_{b,OP}$. Hence, $v_{ce,r} = v_{ce,OP}$ and $i_c(Q_r) = i_{c,OP}$.

Transistor Q_l (on the left) is biased by the controlled voltage source with the same collector-emitter-voltage $U^l_{ce} = U^r_{ce}$, which implies that it will also be biased at the correct current level $i_{c,l} = i_{c,OP}$. Hence, because all other circuits elements are biased at the correct operating point, it follows that $v_{be,l} = v_{be,OP}$ and $i_b(Q_l) = i_{b,OP}$.

For the case when voltage V falls *outside of an operating point*, transistor Q_r is forced in a state with, again, $U^l_{ce} = U^r_{ce}$ trying to force Q_l in the same biasing state. But, this is a contradiction, because the circuit is not at the operating point: $v_{out} = v_{be,l} \neq v_{be,OP}$. Here, again, we consider the two possible cases:

- Case 1: The circuit left of Q_l and connected to its base can deliver more base-current $i_b(Q_l)$ to Q_l than $i_b(Q_r)$, i.e., $i_b(Q_l) > i_b(Q_r)$. This will cause $v_{be,l} > v_{be,r}$, i.e., $v_{out} > v_{in}$.
- Case 2: $i_b(Q_l) < i_b(Q_r)$ and hence $v_{out} < v_{in}$.

In the original circuit (before replacement), $v_{out} > v_{in}$ implies that v_{out} will "pull up" v_{in} to the solution $v_{be,OP}$. By the same argument, $v_{out} < v_{in}$ implies that v_{out} will "pull down" v_{in} to the solution $v_{be,OP}$.

Explanation why the method works

The method is based on the fact that the circuit may be forced in any initial state within the continuous state space. Then, left alone, from each of these initial states the circuit state will converge to one operating point. Hence, one can expect that there are well defined areas of convergence, even though their shape is unknown.

We now consider the state v_{be} on the borderline between two areas of convergence to be a set-value of a closed-loop-control, the actual state being the actual-value. Let the difference between the set-value and the actual-value represent a deviation. If the deviation is positive (negative) the state will increase (decrease). This is the manifestation of a positive feedback. In essence, our method tests whether the FS-transistor under investigation belongs to a positive feedback loop.

In a closed-loop-control system this test is done by cutting the feedback loop and then by considering the open loop, as shown in Fig. 4. This is exactly what we have done by cutting the transistors base connection and by introducing two transistors Q_l and Q_r. A more detailed description of the process is given in [1] (in German).

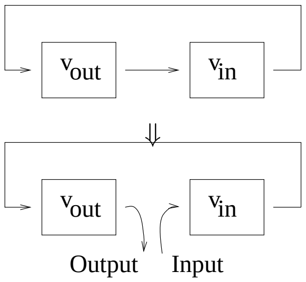

Figure 4: Cutting the feedback-loop

PROBLEM STATEMENT

We have shown in [3] and [4] the equivalence of the replacement in operating points, which is a sufficient condition for detecting operating points in the original circuit: *If* there is an operating point, PFBS method will *then* detected it.

Later, in Section 7, we prove the following:

Theorem 1: *If* the transistor under investigation can cause multiple operating points (all other transistors are supposed to be biased in their correct states), *then* PFBS method will detect *multiple* operating points, all of them being indicated by *different* $v_{out} = v_{in}$.

OUTLINE OF THE PROOF

The proof is based on the fact that it is always possible to replace any branch (except an ideal voltage source) in a given circuit, shown in Fig. 5, by the replacement shown in Fig. 6.

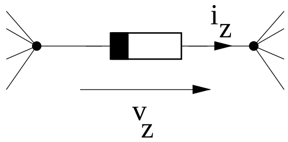

Figure 5: An arbitrary circuit branch (not an ideal voltage source).

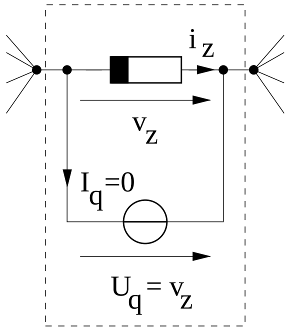

Figure 6: Replacing a branch shown in Fig. 5

In other words, suppose that the circuit is biased at one of its operating points. Then, any branch with the voltage v_z (except an ideal voltage source) can be augmented by an ideal voltage source $U_q = v_z$ connected in parallel to the branch without changing the operating point. (This is also called "soldering iron technique".)
The proof can be easily derived by simply "adding" the characteristics of a nonlinear resistor $f(v_z, i_z) = 0$ and of the voltage source $U_q = v_z$ in i-direction as shown in Fig. 7.

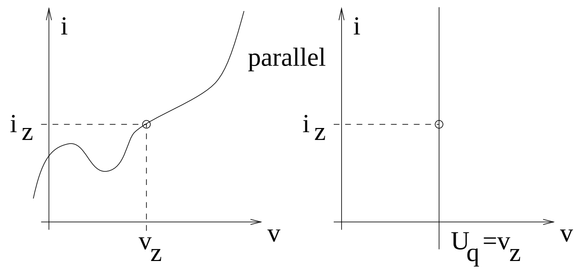

Figure 7: Nonlinear resistor with $f(v_z, i_z) = 0$ and voltage source with $U_q = v_z$ in parallel. The entire characteristic will be equal to $f(v_z, i_z) = 0$ for any *$f(v_z, i_z) = 0$. For a voltage source with $v_z(i_z) = constant$, the branch current i_z is not defined.*

PROOF OF UNIQUENESS

By increasing $v_{in} = v_{be}$ in Fig. 2 or Fig. 3 and by observing $v_{out}(v_{in})$, we can distinguish two operating points only if they have different values for

v_{in}. Hence, a condition to use the PFBS method to reliably indicate multiple operating points, is that different operating points result in different base-emitter-voltages. (Note that other transistors might cause multiple operating points as well. We restrict our arguments to the FS-transistor under investigation.)

Theorem 2: If a transistor that is part of a feedback structure causes multiple dc-operating points, then all values of v_{be} are different. Let us assume that there are two operating points, labeled point 1 and point 2. Then

$$v_{be,1} \neq v_{be,2} \tag{1}$$

holds.

Proof: We prove the theorem by contradiction. Let us assume that

$$v_{be,1} = v_{be,2} \tag{2}$$

are base-emitter voltages of two *distinct* operating points.

If the assumption was true, we could introduce an additional voltage source $U_q = v_{be,1} = v_{be,2}$ in parallel to the base-emitter-voltage without altering either of the operating points. But, because the voltage source U_q would destroy the feedback structure this implies, based on the results of Nielsen and Willson [8], that multiple operating points are impossible. Hence, by contradiction, we conclude that the assumption $v_{be,1} = v_{be,2}$ cannot hold. Hence, $v_{be,1} \neq v_{be,2}$ always holds for a FS-transistor causing multiple operating points. QED.

Using results given in [3] and [4], proof of Theorem 1 follows immediately.

In summary, simply by observing dependence of v_{out} vs. v_{in}, we can distinguish all operating points caused by the FS-transistor under investigation.

CONCLUSIONS

In this paper, we discussed a method designed to help finding multiple operating points of a transistor circuits, and we presented a proof that the method can detect transistors that cause multiple circuit's dc-operating points.

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