PARAMETER EMBEDDING METHODS FOR FINDING DC OPERATING POINTS OF TRANSISTOR CIRCUITS

Wolfgang Mathis, Ljiljana Trajković*, Martin Koch, and Uwe Feldmann**

Department of Electrical Engineering
University of Wuppertal
Fuhlrottstr. 10, D-42097 Wuppertal, Germany
Tel.: +49-202-439-3008, Fax: +49-202-439-3040
E-mail: wolfgang@wein02.elektro.uni-wuppertal.de
* Bellcore, 445 South Street, Morristown, NJ 07960, USA
** Siemens AG, Otto-Hahn-Ring 6, D-8000 München 83, Germany

Abstract

We address the calculation of dc operating points of nonlinear circuits by using parameter embedding methods (also known as continuation and homotopy methods), and we show that the usefulness of these methods depends on the type of a circuit's descriptive equations. We implemented homotopy algorithms in the industrial circuit simulator TITAN. We demonstrated that homotopy methods can be used as an alternative to the Newton-Raphson-type solvers and that they can be successfully applied to solving nonlinear circuit equations as well as to calculating dc operating points of nonlinear transistor circuits.

1 Introduction

In this paper we address the calculation of dc operating points of nonlinear circuits by means of parameter embedding methods (also known as continuation and homotopy methods). One of the first implementations of continuation methods in an industrial circuit simulator was by Cermak in 1971 [1]. Other implementations followed [2]–[6]. Continuation and homotopy methods have been rediscovered several times in circuit theory (see [5] and [7] for extensive reference lists).

Nevertheless, information about implementations of homotopy methods in industrial circuit simulators is not readily available in the literature with the exception of [3]–[6]. In [5], the authors describe an implementation that is suitable for bipolar transistor circuits based on the Gummel-Poon bipolar transistor model [8] with an embedded continuation parameter. In [6], the authors discuss the application of publicly available software packages with continuation algorithms PITCON [9] and HOMPACK [10] for calculation of dc operating points. Although interesting results were obtained, the authors did not address the details of parameter embedding that are crucial for a successful application of homotopy methods for solving circuit equations by circuit simulators.

2 Choice of Equations

We now show that the usefulness of parameter embedding method depends on the type of a circuit's descriptive equations. If a resistive circuit is described by a system of nonlinear equations

$$\mathbf{f}(\mathbf{x}) = \mathbf{0} \tag{1}$$

then, the zeros of this system of equations are its dc operating points. The essential idea of the homotopy approach is to embed a parameter into Equation (1) and, thus, obtain a parameterized family of equations

$$\mathbf{F}(\mathbf{x}, \lambda) = \mathbf{0}, \quad 0 \le \lambda \le 1. \tag{2}$$

The original equation is obtained by setting $\lambda = 1$: $\mathbf{F}(\mathbf{x}, 1) = \mathbf{f}(\mathbf{x}) = \mathbf{0}$. If $\lambda = 0$, then the equation

$$\mathbf{F}(\mathbf{x}, 0) = \mathbf{0} \tag{3}$$

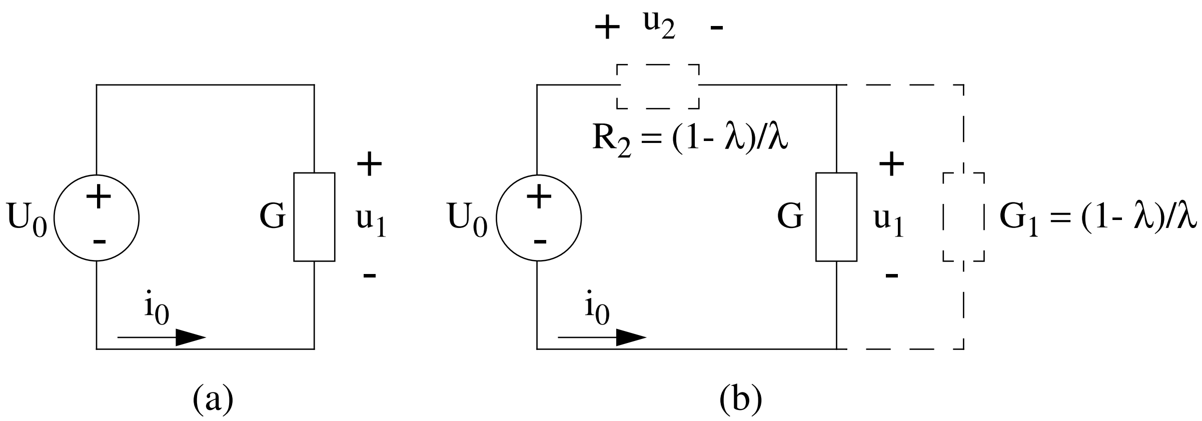

Figure 1: A simple example network for applying MNA equations. (a) Original circuit. (b) Circuit to be solved after parameter embedding.

should be much simpler than the original Equation (1) and finding its zeros should be an "easy" task.

We show that the so-called "standard embedding"

$$\mathbf{F}(\mathbf{x}, \lambda) := (1-\lambda)\mathbf{x} + \lambda \mathbf{f}(\mathbf{x}) = \mathbf{0} \tag{4}$$

fails if we use the well-known Modified Nodal Analysis (MNA) equations [11]. The MNA method is used today in most SPICE-like circuit simulators [12]. The main reason for choosing the MNA approach when writing circuits' equations is to include network elements that are not current sources or do not have an admittance representation (such as certain controlled sources). For such elements one cannot introduce their currents into the vector of network variables. Therefore, in contrast to the basic nodal equations, not all MNA equations represent sums of currents. Some of these equations describe the inter-connection of network elements.

Consider the simple circuit shown in Figure 1(a). The MNA equations of this circuit (see e.g., [13]) are:

$$\begin{pmatrix} G & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ i_0 \end{pmatrix} = \begin{pmatrix} 0 \\ U_0 \end{pmatrix}. \tag{5}$$

In order to apply the homotopy approach, we define the following function:

$$\mathbf{f}(\mathbf{x}) := \begin{pmatrix} G & 1 \\ 1 & 0 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 0 \\ U_0 \end{pmatrix}, \tag{6}$$

where $\mathbf{x} = \begin{pmatrix} u_1 \\ i_0 \end{pmatrix}$. Standard embedding of (6) into (4) leads to the difficulties we are addressing in this paper. Even though we use a simple circuit to illustrate the cause of the problem, it is easy to extend this result to more complex circuits. If, when embedding a parameter, we add to these equations the term $(1-\lambda)\mathbf{x}$ in (4), we create a new circuit. The following equations are obtained by applying the standard embedding, after dividing both equations by λ:

$$\begin{aligned} \frac{1-\lambda}{\lambda} u_1 + (G u_1 + i_0) &= 0 \\ \frac{1-\lambda}{\lambda} i_0 + (u_1 - U_0) &= 0. \end{aligned} \tag{7}$$

A circuit interpretation of the additional terms in (7) leads to the circuit shown in Figure 1(b). It is evident from the polarity of u_2 that $\frac{1-\lambda}{\lambda}$ should be negative. That implies that in the process of finding solutions of (5) we need to find operating points of circuits with negative resistors. Therefore, for some λ, the coefficient matrix of the MNA equations may become singular. In our example, for $G = 1/2$, the coefficient matrix

$$\begin{pmatrix} \frac{1-\lambda}{\lambda} + 1/2 & 1 \\ 1 & \frac{1-\lambda}{\lambda} \end{pmatrix} \tag{8}$$

Simulation Performance			
Circuit	# of Equations	# of Jacobian Eval.	Algorithm
Bipolar Op-Amp	30	508	fixpNf
		187	fixpQf
Bipolar Op-Amp 741 (1)	39	325	fixpQf
Bipolar Adder	449	473	fixpNf
		173	fixpQf
CMOS Reference Circuit	27	66	fixpNf
CMOS Master-Slave Flip-Flop FD2.15.F (1)	47	128	fixpQf
CMOS Op-Amp OP3V3 (1)	55	59	fixpQf
Small MOS Circuit (2)	6	50004	fixpNf
		49993	fixpQf

Table 1: Circuits successfully simulated by the new TITAN dc operating point solver. (1) HOMPACK's fixpNf algorithm failed. (2) TITAN's Newton method failed.

becomes singular for $\lambda \approx 0.562$. This is in contrast to the behavior of nodal equations, where similar embedding always leads to circuits with non-negative resistances [14].

In conclusion, the embedding of a parameter should be done in a manner that avoids non-physical network instances (e.g., occurrence of negative resistors). For that reason, we opted to implement the embeddings suggested by Trajković et al., [3]–[5] and by Hasler [16] (see also Mathis [17]) in an industrial circuit simulator [15].

3 Implementation of a Homotopy Algorithm

All analog circuits that were successfully simulated in [5] and [6] were analog designs with bipolar transistors. We have generalized the embedding method to MOS transistor circuits and have developed an embedding model for MOS transistors. The implementation was done in the industrial circuit simulator TITAN [15] using HOMPACK [10]. We have also added a modification to TITAN for calculating transfer curves.

Simulation performance in terms of the number of Jacobian evaluations of several industrial circuits is shown in Table 1. In all cases, the embedding suggested in [3]–[5]

$$\mathbf{F}(\mathbf{x}, \lambda) := \mathbf{L}(\mathbf{x}) + \mathbf{N}(\lambda \mathbf{x}) = \mathbf{0} \tag{9}$$

was more robust than the embedding approach given in [16]

$$\mathbf{F}(\mathbf{x}, \lambda) := \mathbf{L}(\mathbf{x}) + \lambda \mathbf{N}(\mathbf{x}) = \mathbf{0}. \tag{10}$$

For example, in the case of the CMOS reference circuit for a 16 Mbit DRAM with 27 equations, the embedding (9) required only 66 Jacobian evaluations (with HOMPACK's fixpNf algorithm), while embedding (10) required 67879 Jacobian evaluations. Other examples were even more favorable.

During our simulations, we observed difficulties with choosing the starting points for the homotopy paths. Further research is needed to design a tool that can be easily used by circuit designers.

4 Conclusions

We show that the usefulness of the parameter embedding methods depends on the type of a circuit's descriptive equations. By carefully choosing the embedding equation, we demonstrated that homotopy methods can be successfully applied to solving nonlinear equations and to calculating dc operating points of nonlinear transistor circuits. These methods can be used as an alternative to the Newton-Raphson-type solvers.

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