STAT 201

Some Final Examination Questions

1. A researcher suspects that male university students tend to commute longer distances than female students to university. The researcher draws a simple random sample of 25 female students and an independent simple random sample of 36 male students. The females commute an average of 4 miles with an SD of 6 miles while the males average 6 miles with an SD of 7 miles. The researcher calculates a two sample z-test as follows. The SE for the women's average is estimated to be $6/\sqrt{25} = 1.2$ while the SE for the men's average is estimated to be $7/\sqrt{36} = 1.17$. The SE for the difference is estimated to be $\sqrt{(1.2)^2 + (1.17)^2} = 1.67$. The researcher then computes the test statistic $z = (6-4)/1.67 = 1.19$. and gets $P = 11.5\%$ from the normal tables. The researcher then concludes that men and women commute the same distance to work. Is this conclusion warranted? If not say why not.

   I am looking here for understanding that accepting the null hypothesis is not the same as concluding that it is true, particularly in small samples. We merely don't have much evidence of a difference. Note that in this course you might have been tempted to look at t-tables on 24 degrees of freedom (option 1). If I were asking the question on the final I would have set it up as a two sample t not z test.

2. The pH of a lake (a measure of its acidity) is measured 10 times. The measurements average 4.6 with an SD of 0.22. Find a 90% confidence for the true pH of the lake.

   Use t on 9 df. Comment on the needed assumption that the population of possible measurements follows the normal curve.

3. A simple random sample of 225 British Columbians shows that 100 support the Meech Lake Accord. An independent survey of 400 residents of Québec found 220 who felt that way. Do the residents of the two provinces have different attitudes on this issue?

   This is a two sample problem testing $p_1 = p_2$ where the p_i are the proportions of British Columbians and Quebeckers who support Meech.

The alternative is two sided and you use the z test on page 501. Comment on the fact that you can ignore the impact of sampling without replacement so that the Binomial methods are ok – the population sizes are much large than these sample sizes.

4. For the data in the previous question find an 60% confidence interval for the percentage of residents of British Columbia who support the Meech Lake accord.

   This is a one sample confidence interval for a proportion. See page 476. Again comment on neglecting the sampling without replacement.

5. A gas company inspects its gas meters every 7 years. In a certain town there are 40000 meters. The town's meters are judged acceptable if fewer than 10% of the 40000 rods make an error in measuring gas consumption more than some specified level. In order to see whether or not all 40000 meters will have to be adjusted a sample of 100 meters is tested. Assuming that in fact the town's meters are just barely acceptable–that is, that 4000 of the 40000 meters are too inaccurate–what is the chance that 14 or more of the 100 sampled meters fail the accuracy test?

   This is exactly like the question on the midterm. You have a Binomial with $n = 100$ *and* $p = 0.10$ *and want to get the chance of 14 or more. Convert 13.5 to standard units and look up an area under the normal table to the right of 13.5.*

6. A sample of 400 patients with high serum cholesterol is divided into 200 pairs in such a way that in each pair the two patients have similar medical backgrounds–same sex, similar ages, similar personal habits and soon. For each pair a coin is tossed to see which patient will receive a new drug and which will receive a placebo. In 120 of the pairs the treated patient has a lower serum cholesterol than the control patient. Does the new drug work?

   This would be hard on a final. It is a Binomial problem with $n = 200$ *in which we must test the hypothesis that* $p = 0.5$ *against the alternative* $p > 0.5$ *where* p *is the chance that in a pair the treated patient has a lower serum cholesterol level than the control patient. Use the* z *test from Chapter 18.*

7. A large orchard has 40000 trees. Last year they had an average harvest of 250 kilograms of fruit per tree. In an organic farming experiment a simple random sample of 64 of these trees is picked. Each of the 64 trees was covered this year when the orchard is sprayed with pesticide. The yield this year on the 64 trees averages 270 kilograms with a Standard Deviation of 40 kilograms.

   (a) Carry out the arithmetic of a hypothesis test to determine whether or not covered trees have an average yield higher than 250 kilograms per tree.

   This is a one sample t test (or a z test since the sample size is pretty big. Use 63 df. The parameter of interest this year is μ the average yield of all 40000 trees if they had all been treated. The null is $\mu = 250$ and the alternative is $\mu > 250$ so the test is one tailed.

   (b) The orchardist makes the calculation in a) and concludes that covering all 40000 trees would have raised the total yield. Criticize this conclusion. Be specific.

   This is quite dangerous. First it ignores year to year variation in the yield to be expected due to weather. If this year's weather is just better it may be that this explains the situation for the covered trees. Too, the unsprayed trees derive some benefit, presumably, from the fact that their neighbours were sprayed. If everyone you meet is vaccinated against some disease there is little need for you to be vaccinated.

8. Looking more closely at the data in the previous question the orchardist goes through records for last year for the yields of the 64 sample trees. The figures for last year average 260 kilograms with an SD of 50kg. She discovers, however, that those trees producing less than 220 kilograms last year have increased their production by 20 kg each on average while those producing over 280 kg have not increased their yield at all on average. She develops the following explanation of this observation. Low yielding trees benefit from a year without pesticides. These trees are sensitive to pesticides and their yields have been driven down over the years by repeated application of the pesticides. Identify a pitfall in her reasoning.

This is the regression effect / fallacy.

9. I own a 4 sided die, with sides numbered 1 to 4. You may assume that each of the four sides has the same chance of being chosen when the die is tossed.

   (a) If I toss the die twice what is the chance that the sum of the two throws is 4? Explain carefully, saying clearly where you use the various rules of probability and what assumptions you make.

   Not a suitable question for this course. The answer is 3/16.

   (b) If I toss the die 6 times what is the chance that I get exactly 3 twos?

   This is a Binomial with $n = 6$ and $p = 0.25$. Again not suitable for this course; I was expecting use of the Binomial formula as on page 308.

   (c) If I toss the die 200 times what is the chance that the sum of the tosses is less than 470?

   way too hard for this course.

   (d) If I toss the die 400 times what is the chance that I will get exactly 100 threes.

   This is a normal approximation to a binomial with $n = 400$ and $p = 0.25$. Convert 99.5 and 100.5 to standard units and look up the chance in normal tables.

10. Forty high blood pressure patients are divided into 20 pairs. The pairs are chosen to make the two people in each pair as alike as possible in terms of condition, sex, weight. In each pair a coin is tossed to decide which patient gets a new drug. The other patient gets a placebo. In 27 of the pairs the patient receiving the drug experiences a larger drop in blood pressure than the patient receiving the placebo. Assess the evidence that the new drug is better than a placebo.

    Just like question 6.

11. The hardness of 3 titanium buttons sampled from a day's production is measured and the results are 121, 124, 127. Find a 95% confidence interval for the average hardness of that day's production of buttons. Explain any assumptions you make.

a one sample t confidence interval with $n = 3$. You need to find $\bar{x} = 124$ and $s = 3$. Use 2 df to find the multiplier.

12. The manufacturer of the titanium buttons the question above claims that the process being used produces buttons whose average hardness is at least 128. Do the data reported in 3) provide convincing evidence of fraud? Your answer will include some calculations.

    this is a t-test of the hypothesis $\mu \geq 128$ against $\mu < 128$. Use t on 2 df.

13. A simple random sample of 1500 adult Canadians shows that 200 think the Prime Minister is doing a good job. Find an 88% confidence interval for the proportion of all adult Canadians who think this.

    A confidence for a single Binomial proportion. Remember to note that sampling without replacement is not a problem because the population size is so large.

14. After the announcement of cold fusion by researchers at the University of Utah many groups conducted experiments to check the claims. Imagine that 100 research groups set up experiments to check the claim. Each group tests the null hypothesis of no extra production of heat at the significance level 2%. In point of fact the null hypothesis is true. There is no cold fusion and no extra production of heat. Estimate the probability that none of the 100 research groups will reject the null hypothesis.

    Like Q 16 on assignment 4. The number of groups rejecting the null hypothesis is Binomial with $n = 100$ and $p = 0.02$. Use a normal approximation (which will not be very accurate at all since np is only 2. You need to convert 0.5 to stndard units. This question is too hard for a final.

15. A pair of dice are tested for fairness by tossing them 600 times. A total of 7 is observed on 130 of the 600 tosses. Do the dice seem to be fair?

    The chance of throwing a 7 on a pair of fair dice is $p_0 = 1/6$. This is a hypothesis testing problem for a Binomial with $n = 600$. The null is $p = 1/6$; the alternative is $p \neq 1/2$. Do a two sided z test. Asking you to compute the chance of a 7, (you need to enumerate all 36 outcomes of throwing a pair of dice), is probably too hard for a final.

16. The speed of light (in a vacuum) is measured 15 times. The 15 measurements have an average of 299720 km/sec and a standard deviation of 120 km/sec. Find a 90% confidence interval for the true speed of light. You may assume that individual measurements are normally distributed with an expected value equal to the true speed of light.

    This is a 1 sample t confidence interval. Use 14 df.

17. A manufacturer sells 500 gram packages of coffee. Federal inspectors select 64 packages at random from a month's production. The selected packages have an average weight of 498.5 grams. The manufacturers defend themselves against a charge of short weight by saying that some variation in packed weight is inevitable. They say they use a machine which puts in an average of 500 grams with a standard deviation of 4 grams and that it was just bad luck that the 64 sampled packages had such a low average weight.

    (a) Assuming that the manufacturers are right what is the chance that 64 randomly sampled packages would have an average weight less than or equal to 498.5 grams?

        Convert 498.5 to standard units by subtracting 500 and dividing by $4/\sqrt{64}$. *Look up the area in normal tables to the left of the result.*

    (b) On the basis of your answer to a) do you believe the manufacturer? Give a 1 sentence explanation.

        The number computed in a) is the P value for a one sided test of $H_0 : \mu \geq 500$ *against the alternative* $\mu < 500$. *It is tiny so the manufacturer is probably wrong.*

18. Of the 64 packages examined in the previous question 40 weigh less than 500 grams. Find a 98% confidence interval for the proportion, p, of the month's production of packages which weigh less than 500 grams.

    This is a one sample confidence interval for a binomial proportion when $n = 64$ *and* $\hat{p} = 40/64$.

19. A simple random sample of 900 adult Quebec voters is drawn. Each is asked how he or she will vote on the referendum.

(a) If 465 say they will vote no and 435 say they will vote yes how strong is the evidence that the referendum will fail?

A test of the hypothesis that $p \geq 0.5$ *against* $p < 0.5$ *where* p *is the population proportion intending to vote for the referendum. You have* $n = 900$ *and* $\hat{p} = 465/900$*. The* P *value is large indicating little evidence against the asssertion that the referendum will pass. Indeed the evidence against* $p \leq 0.5$ *is quite strong. NOTE: you can also do this problem and the rest with p being the proportion who intend to vote against; you have to keep straight which is which.*

(b) Suppose that a week earlier a survey of 1200 voters had given 650 yes and 550 no. Has the proportion of the population who will vote no increased?

A two sample test of the hypothesis $p_{NOW} = p_{Then}$ *against the alternative* $p_{NOW} < p_{Then}$ *where each p is the proportion intending to vote* **for** *the referendum. If you chose p to be the proportion intending to vote* **against** *switch the direction of the inequality in the alternative.*

(c) Briefly (2 or 3 sentences) criticize the assumptions used in the previous two parts and comment on the realism of the description of outcome of the survey.

20. Government offices maintain standard weights, whose weights are known very accurately, in order to compare other weights and keep scales well standardized. One of these weights is weighed 10 times giving an average of 496 micrograms over 10 grams; the SD of these measurements is 30 micrograms.

(a) Give a 90% confidence interval for the true weight explaining your assumptions.

A one sample t interval with 9 df.

(b) The weight is then dropped accidentally and weighed 6 further times, averaging 465 micrograms over 10 grams with an SD of 25 micrograms. Did the weight change when it was dropped?

A two sample t test with a two sided alternative. Use 5 degrees of freedom with option 1 (or 15 with the pooled estimate of σ *and option 3. On a final use option 1.*

(c) Assume that the scales make errors with an SD of 30 micrograms when weighing objects whose true weight is close to 10 grams. How many times would you have to weigh such a weight to have at least a 95% chance of detecting a difference of 5 micrograms from 10 grams, using a 10% level test.

Beyond the scope of this course; this question dealt with **power**.

21. A simple random sample of 900 adult British Columbians is drawn. Each is asked how much beer he or she drinks per day.

(a) Assume that the 900 answers average 600 ml with a standard deviation of 750 ml. Give a 90% confidence interval for the average daily beer consumption of all adult British Columbians.

A one sample t or z confidence interval. The sample size is so large you would either use 1000 df for the t or normal tables. Either would be ok.

(b) Suppose that a year earlier a survey of 1600 voters had given an average of 550 ml with a standard deviation of 800 ml. Has daily beer consumption increased over the year?

A two sample t test again with many, many df.

(c) In the more recent survey (the one of 900 people) 300 report not drinking anything while in the earlier survey the corresponding figure was 400 people. Has there been a change in the fraction of all adults who do not drink?

A two sample z test for proportions. The test is two sided.

(d) Briefly (2 or 3 sentences) criticize the assumptions used in the previous three parts and comment on the realism of the description of outcome of the survey.

I would be looking for ideas like: response bias – people are sensitive to questions about how much they drink and may not answer honestly, and non-response biases which are present in all real surveys.

22. I have a bag with 3 balls: one Red, one Green, one Blue. I reach into the bag and pick out a ball at random. I also spin a nickel and see if it lands heads (H) or tails (T). The coin has chance 1/4 of landing heads

the way I spin it. Make a list of all the outcomes in the sample space for this experiment and show the probability of each outcome. Explain the rules you used to get these probabilities.

There are 6 outcomes: $\{RH, GH, BH, RT, GT, BT\}$*. The probabilities are 1/12 for the first 3 and 1/4 for the last 3 of these.*

23. A simple random sample of 800 Canadians shows that the correlation between height and weight is 0.8. The sample includes 200 children under the age of 12. If these children are eliminated will the correlation for the remaining 600 people likely be higher than 0.8, around 0.8 or lower than 0.8. Explain with a graph.

    The correlation will go down since you are eliminating the short, light people to a large extent.

24. In a study of 315 adults to investigate the relation between diet and blood levels of beta-carotene the attached scatterplot of daily calories consumed versus grams of fat consumed has the following summary statistics: for calories the mean is 1781 and the sd is 623; for fat the mean is 76.8 and the sd is 33.5. The correlation is 0.9.

    (a) If I drew a simple random sample of 10 of these 315 adults what is the chance that I would get a sample average daily fat consumption of more than 95 g?

    Make a normal approximation. Convert 95 to standard units by subtracting 76.8 and dividing by $33.5/\sqrt{10}$*. Then look up the area to the right under the normal curve.*

    (b) In the same study the researchers measured the amount of fiber each adult consumed per day (in grams). Is the correlation between this measurement and daily calorie consumption likely to be positive, near 0 or negative. Your answer will need a short simple explanation (and you don't need to know anything sophisticated about dietary fiber and so on). In grading I will be judging whether or not the explanation you advance predicts the answer you give to the main question.

    The most obvious theory is that the correlation will be positive because people who eat more will eat more of everything.

(c) In the same study the authors measured daily cholesterol consumption; see the attached histogram. The mean was 242 milligrams and the sd was 132 milligrams . Approximately what percentage of the adults consumed more than 450 milligrams of cholesterol per day. You should make the approximation and comment on its appropriateness.

This is a normal approximation. Convert 450 to standard units by subtracting 242 and dividing by 132. You need to comment on whether or not the histogram looks reasonably normal.

(d) Predict the daily fat consumption of an adult who consumes 3000 calories per day.

Make a regression predictions. y *is daily fat consumption and* x *is daily calories.*

25. If I spin the nickel in problem 1 eight hundred times what is the chance that I will get exactly 200 heads? WARNING: I will not accept exact answers found by using a calculator; I want an approximation.

    Normal approximation to Binomial. $n = 800$, $p = 0.25$. *Convert 199.5 and 200.5 to standard units and look up area.*

26. In a study of 1000 Canadian families it is found that there are 1980 children. An investigator goes through hospital records to discover the birth weights of these babies. S/he plans to treat the number of these babies whose birth weight is under 6 pounds as having a Binomial distribution. Is this wise? Explain your answer clearly.

    Not a srs of 1980 children so the binomial distribution won't be appropriate.

27. In a study of the relation between education levels of husbands and wives a sample is drawn and the correlation is found to be 0.5. For a husband with 16 years of education the regression method predicts that the wife will have 14 years of education. You may assume that both these figures are above average. For a wife with 14 years of education will the prediction for husbands be more than 16, 16 or less than 16 years of education? The quality of your explanation is important here.

    I am looking for a discussion of the regression effect. The answer is less than 16.

28. You want to know if two populations of raccoons have the same size distributions. You collect 11 animals from one and 8 from the other. The first sample has mean weight 4.2 kg and SD 4.68 kg; the second sample has mean weight 5.6 kg and SD 3.92 kg. Are the population mean weights different?

    Two sample t test. Use option 1.