STAT 270 Lecture 19
Fall 2015
23 October 2015

- I covered up to slide 16 of "Continuous distributions".
- I got well in to Section 5.2 in the text.
- Problems from text: see the next lecture.
- I defined the Normal(μ, σ^2) density.
- I worked out the corresponding cdf in terms of the standard normal cdf Φ.
- I defined the standard normal distribution.
- We have covered up to the first bit of Section 5.2 in the text.
- Handwritten slides.
- Key jargon, ideas:
  - The Normal(μ, σ^2) density is
  $$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-z^2/2).$$
  - If X is Normal(μ, σ^2) then $Y = aX + b$ is Normal($a\mu + b, a^2\sigma^2$).
  - The expected value for a standard normal density is 0.
  - I proved that the standard normal density is density. That is, I proved that
  $$I = \int_{-\infty}^{\infty} e^{-x^2/2}\, dx = \sqrt{2\pi}$$
  - To do so I showed $I^2 = 2\pi$. Remember
  $$I = \int_{-\infty}^{\infty} e^{-y^2/2}\, dy.$$
  - So
  $$I^2 = \int_{-\infty}^{\infty} e^{-x^2/2}\, dx \int_{-\infty}^{\infty} e^{-y^2/2}\, dy$$
  $$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(-(x^2 + y^2)/2\, dx\, dy$$

– This integral can be done in polar co-ordinates:

$$x = r\cos(\theta)$$

and

$$y = r\sin(\theta).$$

The Jacobian is

$$dx\,dy = r\,dr\,d\theta.$$

In polar co-ordinates the plan corresponds to

$$0 \le \theta < 2\pi$$

and

$$0 < r$$

so

$$I^2 = \int_0^\infty \int_0^{2\pi} \exp(-r^2/2) r\,d\theta\,dr.$$

The inside integral, over θ, gives 2π so

$$I^2 = 2\pi \int_0^\infty r\exp(-r^2/2)\,dr.$$

Since

$$\frac{d}{dr}\exp(-r^2/2) = -r\exp(-r^2/2)$$

we find

$$I^2 = -\,2\pi\exp(-r^2/2)\Big|_0^\infty = 2\pi.$$