STAT 350

Assignment 2

NOTE: Due by 3 PM Friday 23 May in my mailbox in the Statistics and Actuarial Science Department or by email to lhzhao@cs.sfu.ca by 8PM Friday.

The next two questions require the use of computing software. Those of you who have not done any computing should come to the tutorial next week in the PC lab – room details to follow. You may use any statistical package you like. In class I will use SAS, JMP or R as I see fit.

1. From the text questions 1.19 and 1.23. The data are available on the disk accompanying the text; email me if you can't get it. In addition: write a short paragraph discussing the difficulties in using data on admitted students to study the relation between ACT score and first year GPA.

2. From the text questions 2.13 a and b and 2.23 a, b and c. In 2.23 c give a P-value and interpret this P-value.

3. Working with partitioned matrices. Suppose that the design matrix X is partitioned as $X = [\mathbf{1}|X_1|X_2]$ where X_i has p_i columns.

   (a) Write $X^T X$ as a partitioned (3 rows, 3 columns) matrix.

   (b) A matrix
   $$A = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix}$$
   where each A_i is a square matrix is called *block diagonal*. Show that A^{-1} exists if and only if each A_i^{-1} exists and that then A^{-1} is block diagonal.

   (c) Suppose that $\mathbf{1}^T X_i = 0$ for $i = 1, 2$ and $X_1^T X_2 = 0$. Show that $X^T X$ is block diagonal and give a formula for $(X^T X)^{-1}$.

   (d) Suppose $\beta^T = [\beta_0|\beta_1^T|\beta_2^T]$ is partitioned to conform with the partitioning of X (that is β_0 is a scalar and β_i is a column vector of length p_i for $i = 1, 2$. Let $\tilde{\beta}_0$ be obtained by fitting
   $$Y = \mathbf{1}\beta_0 + \epsilon$$

by least squares, $\tilde{\beta}_1$ be obtained by fitting

$$Y = X_1\beta_1 + \epsilon$$

and similarly for $\tilde{\beta}_2$. Let $\hat{\beta}$ be the usual least squares estimate for

$$Y = X\beta + \epsilon\,.$$

Show that $\hat{\beta}^T = [\tilde{\beta}_0|\tilde{\beta}_1^T|\tilde{\beta}_2^T]$.

(e) Let $\hat{\mu}_i$ be the vectors of fitted values corresponding to the estimates $\tilde{\beta}_i$ for $i = 0, 1, 2$. Show that for $i \neq j$ we have $\hat{\mu}_i \perp \hat{\mu}_j$.

(f) For the design matrix X_b of the first assignment identify X_1 and X_2 and verify the orthogonality condition of this problem.

DUE: Friday, 23 May.