STAT 350

Assignment 5

NOTE: Due by 3 PM Friday 18 July in my mailbox in the Statistics and Actuarial Science Department or by email to lhzhao@cs.sfu.ca by 8PM Friday.

1. Suppose the two cars in our mileage problem are of the same make but that one vehicle was equipped with a special pollution control device. In 4 or 5 sentences comment on the experimental design as a method of determining whether or not the device reduces emissions and on what else you would want to find out from the experimenter to help interpret the results.

2. In this question you will derive some of the formulas for case deleted statistics. Suppose that X is an $n \times p$ design matrix. Let x_i^T be the i^{th} row of X (so that, x_i is a column vector or dimension p). Let $X_{(i)}$ be the design matrix with case i deleted and $Y_{(i)}$ the vector of $n-1$ responses with case i deleted.

   (a) Use partitioned matrices to show
   $$X_{(i)}^T Y_{(i)} = X^T Y - x_i Y_i \, .$$

   (b) Show
   $$X_{(i)}^T X_{(i)} = X^T X - x_i x_i^T \, .$$

   (c) Suppose B is an invertible symmetric $p \times p$ matrix and v is a column vector of dimension p. Show, by direct multiplication, that $(B - vv^T)^{-1}$ is of the form
   $$B^{-1} + rB^{-1}vv^T B^{-1}$$
   and give a formula for the scalar r.

   (d) Apply to previous part to show
   $$(X_{(i)}^T X_{(i)})^{-1} = (X^T X)^{-1} + r_i (X^T X)^{-1} x_i x_i^t (X^T X)^{-1}$$
   and give a formula for r_i in terms of the leverage $h_{ii} = x_i^T (X^T X)^{-1} x_i$.

(e) Show $1 + h_{ii} r_i = r_i$.

(f) Show that

$$\begin{aligned}\hat{\beta}_{(i)} &= \hat{\beta} + r_i (X^T X)^{-1} x_i x_i^T \hat{\beta} - (1 + h_{ii} r_i)(X^T X)^{-1} x_i Y_i \\ &= \hat{\beta} - r_i (X^T X)^{-1} x_i \hat{\epsilon}_i \,.\end{aligned}$$

(g) Deduce that $\hat{\mu}_{(i)} = \hat{\mu}_i - r_i h_{ii} \hat{\epsilon}_i$.

(h) Show that the i^{th} PRESS residual $Y_i - \hat{\mu}_{(i)}$ is given by

$$Y_i - \hat{\mu}_{(i)} = r_i \hat{\epsilon}_i$$

(i) Derive the formula for the i^{th} externally studentized (case deleted) residual.

3. Problem 22.11 parts a, b anc c, 22.12 part c.

4. Problem 24.12, 24.13 parts c,d,e,f,g only and 24.14. Note that 24.12 has only parts d and e in the text.

5. Question 10.7 and 10.11 parts a, b, d, and f.

6. In this problem you will prove that

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \qquad -\infty < x < \infty$$

is a density.

(a) Let $I = \int_{-\infty}^{\infty} \phi(x) dx$. Show that

$$I^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(x)\phi(y)\, dx\, dy.$$

HINT: What is $\int_{-\infty}^{\infty} \phi(y) dy$ in terms of I.

(b) Now if

$$J = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \frac{1}{\sqrt{2\pi}} e^{-y^2/2}\, dx\, dy$$

do the double integral J in polar co-ordinates ($x = r\cos\theta$, $y = r\sin\theta$) to show $J = 1$.

(c) Deduce that ϕ is a density.