STAT 380 Lecture 18

Summary of Markov Process Results

- Chapman-Kolmogorov equations:

$$\mathbf{P}_{ik}(t+s) = \sum_j \mathbf{P}_{ij}(t)\mathbf{P}_{jk}(s)$$

- Exponential holding times: starting from state i time, T_i, until process leaves i has exponential distribution, rate denoted v_i.

- Sequence of states visited, $Y_0, Y_1, Y_2, \ldots$ is Markov chain – transition matrix has $\mathbf{P}_{ii} = 0$. Y sometimes called **skeleton**.

- **Communicating classes** defined for skeleton chain. Usually assume chain has 1 communicating class.

- Periodicity irrelevant because of continuity of exponential distribution.

- Instantaneous transition rates from i to j:

$$q_{ij} = v_i \mathbf{P}_{ij}$$

- Kolmogorov backward equations:

$$\mathbf{P}'_{ij}(t) = \sum_{k \neq i} q_{ik}\mathbf{P}_{kj}(t) - v_i \mathbf{P}_{ij}(t)$$

- Kolmogorov forward equations:

$$\mathbf{P}'_{ij}(t) = \sum_{k \neq j} q_{kj}\mathbf{P}_{ik}(t) - v_i \mathbf{P}_{ij}(t)$$

- For strongly recurrent chains with a single communicating class:

$$\mathbf{P}_{ij}(t) \to \pi_j$$

- Stationary initial probabilities π_i satisfy:

$$v_j \pi_j = \sum_{k \neq j} q_{kj}\pi_k$$

- Transition probabilities given by

$$\mathbf{P}(t) = e^{\mathbf{R}t}$$

where $\mathbf{R}$ has entries

$$\mathbf{R}_{ij} = \begin{cases} q_{ij} & i \neq j \\ -v_i & i = j \end{cases}$$