STAT 380 Lecture 24

Simulation

Monte Carlo computation of expected value:

To compute $\mathrm{E}(X)$: do experiment n times to generate n independent observations all with same distribution as X.

Get $X_1, \ldots, X_n$.

Use

$$\bar{X} = \frac{1}{n} \sum_1^n X_i$$

as an estimate of $\mathrm{E}(X)$

To estimate $\mathrm{E}(g(X))$: use same X_i and compute

$$\frac{1}{n} \sum_1^n g(X_i) \, .$$

Random Number Generation

In practice: random quantities are not used.

Use: **pseudo-random** uniform random numbers.

They "behave like" iid Uniform(0,1) variables.

Many, many generators. One standard kind: **linear congruential generators**.

Start with x_0 an integer in range $0 \le x_0 < m$. Compute

$$x_{n+1} = a x_n + b \mod m$$

Here the **mod** means compute the remainder after division by m.

Integers a and b are chosen so that

- the sequence $x_0, \ldots, x_{m-1}$ runs through the set $\{0, \ldots, m-1\}$ (or as much of it is possible, sometimes).
- the sequence of pairs
  $$(x_0, x_1), (x_1, x_2), \ldots, (x_{m-1}, x_m)$$
  fills up as much of the square $\{(i,j) : 1 \le i, j \le m-1\}$.
- similarly for triples (x_k, x_{k+1}, x_{k+2}).

Use

$$U_n = \frac{x_n}{m}$$

as Uniform(0,1) random numbers.

General Random Variables

We now pretend we can generate $U_1, U_2, \ldots$ which are iid Uniform(0,1).

Monte Carlo methods for generating a sample:

- Transformation:
  - Inverse Probability Integral Transform
  - Multivariate (e.g. Box Müller)
- Acceptance Rejection
- Markov Chain Monte Carlo