STAT 380

Midterm Examination 1

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Instructions: This is an open book exam. You may use notes, books and a calculator. The exam is out of 20. Each part of each question is worth 5. I think the b) parts can be done without doing either of the a) parts. Be as clear as possible about what you are doing but if you can only give an intuitive explanation for your calculations please do give it.

1. Suppose you toss a fair die 7 times. Let N be the number thrown on the first toss and $Y_1, \ldots, Y_6$ be the numbers thrown on the next 6 tosses. Let T be the total of the first toss and the first N of the Y's.

   (a) Show that $\mathrm{E}(T|N) = 4.5N$ and that $\mathrm{Var}(T|N) = 31N/12$.

   (b) Use the results of a) to compute the mean and variance of T.

2. A coin is tossed until the first Head is observed. Let N be the number of tosses and assume that the probability of Heads on a single toss is p. Find $\mathrm{E}(\sin(\pi N/2))$ as follows.

   (a) Let $t = \mathrm{E}(\sin(\pi N/2))$ and $u = \mathrm{E}(\cos(\pi N/2))$. Derive the equations

   $$t = p\sin(\pi/2) + (1-p)(\sin(\pi/2)u + \cos(\pi/2)t)$$

   and

   $$u = p\cos(\pi/2) + (1-p)(\cos(\pi/2)u - \sin(\pi/2)t).$$

   (b) Solve the equations in a).

   Note: you may want to use

   $$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$$

   $$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

   $$\sin(\pi/2) = 1$$

   and

   $$\cos(\pi/2) = 0.$$