STAT 380

Midterm Examination

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Instructions: This is an open book exam. You may use notes, books and a calculator. The exam is out of 25, 5 marks per question. I will be marking for clarity of explanation as well as correctness.

1. Consider the following strategy for comparing two medical treatments, say treatment A and treatment B. Patients are treated one at a time and the result of each treatment is recorded as a Success or a Failure. Every time a treatment succeeds the next patient is treated with the same treatment which was just successful. When a treatment fails, the next patient is treated with the other treatment. Suppose that the probability that treatment A succeeds is p_A while the probability that treatment B succeeds is p_B. In the long run what fraction of patients are treated with treatment B?

2. With the same set up as in the first question suppose that the treatment is changed only after two consecutive failures. Using the four states:

   0 About to use A, last trial was not a failure with A.

   1 About to use A, last trial was a failure with A.

   2 About to use B, last trial was not a failure with B.

   3 About to use B, last trial was a failure with B.

   give the transition matrix of a Markov Chain which can be used to determine what fraction of patients are treated with treatment B in the long run and show clearly what equations you would solve to find the answer. You need not actually solve the equations but I don't want the equations left in matrix form.

3. A Markov Chain has state space $\{1, 2, 3, 4\}$ and transition matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Identify all the communicating classes and say whether or not each is transient.

(b) Let $q_k = \mathrm{P}(\text{there is an } n \text{ such that } X_n = 4 | X_0 = k)$. Derive the equations

$$q_2 = (q_2 + q_3)/3 \text{ and } q_3 = (q_2 + 2q_3 + 1)/4$$

4. Cosmic rays are detected by a particle detector according to a Poisson Process with rate 2. An arriving cosmic ray is detected with probability 0.8 independent of all other cosmic rays and the time at which the cosmic ray arrives. Compute the probability that 4 cosmic rays arrived in a time period of length $t = 5$ starting from time $t = 0$ given that given that 8 cosmic rays were detected in the time period from $t = 0$ to $t = 8$.

Extra space