Name: ______________ **Student Number**: ______________

STAT 380: Spring 2018

Midterm Examination # 2

Richard Lockhart 9 March 2018

Instructions: This is a closed book exam. You are permitted to use 2 sheets of notes, machine-written or hand-written. You may use both sides of the sheets and I place no limits on font size. Calculators are not permitted nor are any other electronic aids. The exam is out of 30. Please put your name and student number on the front page and on the grade sheet page at the end. You should have 10 pages; this cover page, 6 pages of questions, 2 pages of extra space, and the last page is a grade sheet. I will be marking for clarity of explanation as well as correctness. Without a clear explanation you should not expect to get more than half marks. Remember I warned you after the first midterm.

1. Suppose that X and Y are independent random variables. Assume that X has an $\mathrm{Exp}(\lambda_x)$ distribution and Y has an $\mathrm{Exp}(\lambda_y)$ distribution.

   (a) What is $P(Y > X|X = x)$? [3 marks]

   (b) What is $P(Y > X|X)$? [3 marks]

   (c) Write $P(X < Y)$ as a double integral and show that this probability is

$$\frac{\lambda_x}{\lambda_x + \lambda_y}.$$

[3 marks]

(d) Now suppose $X_1, \ldots, X_n$ are a sample from the Exponential(λ_x) distribution and $Y_1, \ldots, Y_m$ are a sample from Exponential(λ_y) distribution. Find

$$P\left(\min\{X_1, \ldots, X_n\} < \min\{Y_1, \ldots, Y_m\}\right).$$

[5 marks]

(e) A testing lab compares two types of machine. Type A machines run for an exponentially distributed amount of time with mean 5 hours; Type B machines also have exponentially distributed running times with mean 8 hours. If the lab starts n Type A machines and m Type B machines at the same instant what is the chance that the first two machines to stop running are both Type A machines. [4 marks]

2. In a certain industry accidents occur according to a Poisson process with rate λ equal to $1/3$ per day.

   (a) There are 30 days in June. What is the chance that there are no accidents after June 20? (No need to get out a calculator; just give me the simplest formula you can.) [4 marks]

(b) Given that there are 15 accidents in June what is the chance there are no accidents after June 20? (No need to get out a calculator; just give me the simplest formula you can.) [4 marks]

3. If T has a uniform distribution on $[0, 1]$ what is the hazard rate of T? [4 marks]

Extra space

Extra space

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Grade Sheet

1a		3	1b		3
1c		3	1d		5
1e		4	2a		4
2b		4	3		4

Total		**30**