STAT 801

Problems: Assignment 2

1. Suppose X has the Beta(α, β) density

$$f(x;\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} 1(0 < x < 1)$$

   Find the distribution of $Y = X/(1-X)$.

2. In class I showed

$$f(x_1, x_2) = 24 x_1 x_2 1(0 < x_1) 1(0 < x_2) 1(x_1 + x_2 < 1)$$

   is a density. If (X_1, X_2) has this density what is the distribution of $X_1 + X_2$?

3. Suppose X and Y are iid $N(0, \sigma^2)$.

   (a) Show that $X^2 + Y^2$ and $X/(X^2+Y^2)^{1/2}$ are independent.

   (b) Show that $\Theta = \arcsin(X/(X^2+Y^2)^{1/2})$ is uniformly distributed on $(-\pi/2, \pi/2]$.

   (c) Show X/Y is a Cauchy random variable.

4. Suppose X is Uniform on [0,1] and $Y = \sin(4\pi X)$. Find the density of Y.

5. Suppose X and Y have joint density $f(x, y)$. Prove from the definition of density *given in class* that the density of X is $g(x) = \int f(x, y)\, dy$.

6. Suppose X is Poisson(θ). After observing X a coin landing Heads with probability p is tossed X times. Let Y be the number of Heads and Z be the number of Tails. Find the joint and marginal distributions of Y and Z.

7. Let p_1 be the bivariate normal **density** with mean 0, unit variances and correlation ρ and let p_2 be the standard bivariate normal **density**. Let $p = (p_1 + p_2)/2$.

   (a) Show that p has normal margins but is not bivariate normal.

   (b) Generalize the construction to show that there rv's X and Y such that X and Y are each standard normal, X and Y are uncorrelated but X and Y are not independent.

8. Suppose X and Y are independent with $X \sim N(\mu, \sigma^2)$ and $Y \sim N(\gamma, \tau^2)$. Let $Z = X + Y$. Find the distribution of Z given X and that of X given Z.