# STAT 801

## Problems: Assignment 3

1. Suppose $X_1, \ldots, X_n$ are iid real random variables with density $f$ . Let $X_{(1)}, \ldots, X_{(n)}$ be the $X$'s arranged in increasing order.

   (a) Find the joint density of $X_{(1)}, \ldots, X_{(n)}$ .

   (b) Suppose $f = 1_{[0,1]}$. Prove that $(X_{(1)}/X_{(k)}, \ldots, X_{(k-1)}/X_{(k)})$ is independent of $(X_{(k)}, \ldots, X_{(n)})$.

   (c) Again with $f = 1_{[0,1]}$ find the density of $X_{(k)}$.

   (d) Again with $f = 1_{[0,1]}$ find the density of $X_{(k)} - X_{(j)}$.

2. Suppose $X_1, \ldots, X_{n+1}$ are iid exponential. Let $S_m = \sum_1^m X_i$.

   (a) Find the joint density of $(X_1/S_{n+1}, \ldots, X_n/S_{n+1})$.

   (b) Find the joint density of $(S_1/S_{n+1}, \ldots, S_n/S_{n+1})$.

3. Suppose $X_1, \ldots, X_n$ are iid N($\mu$,$\sigma^2$). Let $\bar{X}_m = (X_1 + \cdots + X_m)/m$. Let $S_m^2 = \sum_1^m (X_i - \bar{X}_m)^2$.

   (a) Develop a recurrence relation for $S_m$ and $\bar{X}_m$, expressing $S_m$ and $\bar{X}_m$ in terms of $X_m$, $S_{m-1}$ and $\bar{X}_{m-1}$.

   (b) Find the joint density of $(\bar{X}_n, S_2^2, \ldots, S_n^2)$.

   (c) Generate data from N(0,1). By adding $10^k$ to the data for some large values of $k$ compare the numerical performance of these recurrence relations to that of the one pass formula using $T_1 = \sum_1^n X_i$, $T_2 = \sum X_i^2$ and the usual computing formulas for the sample variance.

4. Compute the characteristic function, cumulants and the first 5 central moments for the Poisson($\lambda$) distribution. You may feel free to use MAPLE to help take derivatives or whatever.

5. Compute the characteristic function, cumulants and the first 5 central moments for the Gamma distribution with shape parameter $\alpha$ and scale parameter $\beta$. You may feel free to use MAPLE to help take derivatives or whatever.