STAT 801

Problems: Assignment 4

1. Develop explicit formulas for the saddlepoint approximation to the density of the mean of a sample of size n from the exponential distribution. Compare the results with the true Gamma density.

2. Suppose X, Y and Z are independent standard exponentials. Use *numerical* Fourier inversion of the characteristic function to compute the density of $X + Y/2 + Z/4$ at 1. You may use the splus function *integ.romb* (or any other function) found by attaching the directory

   /home/math4/lockhart/research/software/quadrature/.Data.

   A handout on these functions is available

3. Suppose X is an integer valued random variable. Let $\phi(t)$ be the characteristic function of X.

   (a) Show that
   $$\mathrm{P}(X = k) = (2\pi)^{-1} \int_{-\pi}^{\pi} \phi(t) e^{-itk}\, dt$$

   (b) Suppose further that X is a random variable such that $P(X \text{ is even}) = 1$. Show that for k even
   $$\mathrm{P}(X = k) = \pi^{-1} \int_{-\pi/2}^{\pi/2} \phi(t) e^{-itk}\, dt$$

4. Suppose $X_1, \ldots, X_{2n}$ are independent random variables such that $\mathrm{P}(X_i = 1) = \mathrm{P}(X_i = -1) = 0.5$. Prove that as $n \to \infty$
   $$(2n)^{1/2}\mathrm{P}(X_1 + \cdots + X_{2n} = 0)/\varphi(0) \to 2$$
   where φ is the standard normal density. You should use part b) of the previous problem and Taylor expansion of the characteristic function around 0. Also do the same thing using Stirling's formula.

5. If $(X_1, Y_1), \ldots, (X_n, Y_n)$ are independent bivariate normal random variables find the limiting distribution of $n^{1/2}(r - \rho)$ where r is the sample correlation coefficient and ρ is the population correlation. HINTS: the problem is easier for $\mu_X = \mu_Y = 0$ and $\sigma_X = \sigma_Y = 1$ so prove that you can assume these values for the parameters without loss of generality. Next remember that the correlation coefficient can be computed from the 5 summary statistics $\bar{X}, \bar{Y}, \bar{X^2}, \bar{Y^2}, \bar{XY}$. Use the central limit theorem (multivariate version) to compute an approximate normal distribution for this vector.