STAT 801

Problems: Assignment 6

1. **Postponed from Assignment 5**: Let T_i be the error sum of squares in the ith cell in the first question of Assignment 5.

   (a) Find the joint density of the T_i.

   (b) Find the best estimate of σ^2 of the form $\sum_1^p a_i T_i$ in the sense of mean squared error.

   (c) Do the same under the condition that the estimator must be unbiased.

   (d) If only $T_1, \ldots T_p$ are observed what is the MLE of σ?

   (e) Find the UMVUE of σ^2 for the usual one-way layout model, that is, the model of the last two questions.

2. Suppose $X_1, \ldots, X_m$ are iid $N(\mu, \sigma^2)$ and $Y_1, \ldots, Y_n$ are iid $N(\chi, \tau^2)$. Assume the Xs are independent of the Ys.

   (a) Find complete and sufficient statistics.

   (b) Find UMVUE's of $\mu - \chi$ and σ^2/τ^2.

   (c) Now suppose you know that $\sigma = \tau$. Find UMVUE's of $\chi - \mu$ and of $(\chi - \mu)/\sigma$. (You have already found the UMVUE for σ^2.)

   (d) Now suppose σ and τ are unknown but that you know that $\mu = \chi$. Prove there is no UMVUE for μ. (Hint: Find the UMVUE if you knew $\sigma/\tau = a$ with a known. Use the fact that the solution depends on a to finish the proof.)

   (e) Why doesn't the Lehmann-Scheffé theorem apply?

3. Suppose $X_1, \ldots, X_n$ iid Poisson(λ). Find the UMVUE for λ and for $1 - \exp(-\lambda) = P(X_1 \neq 0)$.

4. Suppose $X_1, \ldots, X_n$ iid with

$$P(X_1 = k) = \text{Prob}(\text{Poisson}(\lambda) = k | \text{Poisson}(\lambda) > 0)$$

   for $k = 1, 2, 3, \ldots$. For $n = 1$ and 2 find the UMVUE of $1 - \exp(-\lambda)$. (Hint: The expected value of any function of X is a power series in λ divided by $e^\lambda - 1$. Set this equal to $1 - \exp(-\lambda)$ and deduce that two power series are equal. Since this implies their coefficients are the same you can see what the estimate must be.)

5. Exponential families: Suppose $X_1, \ldots, X_n$ are iid with density

$$f(x; \theta_1, \ldots, \theta_p) = c(\theta) \exp(\sum_1^p T_i(x)\theta_i) h(x).$$

(a) Find minimal sufficient statistics.

(b) If $S_1, \ldots, S_p$ are the minimal sufficient statistics show that setting $S_i = \mathrm{E}_\theta(S_i)$ and solving gives the likelihood equations. (Note the connection to the method of moments.)