

Statistical Inference

Definition: A **model** is a family $\{P_\theta; \theta \in \Theta\}$ of possible distributions for some random variable X .

WARNING: Data set is X , so X will generally be a big vector or matrix or even more complicated object.)

Assumption in this course: true distribution P of X is P_{θ_0} for some $\theta_0 \in \Theta$.

JARGON: θ_0 is *true value* of the parameter.

Notice: this assumption is wrong; we hope it is not wrong in an important way.

If it's wrong: enlarge model, put in more distributions, make Θ bigger.

Goal: observe value of X , guess θ_0 or some property of θ_0 .

Classic mathematical versions of guessing:

1. Point estimation: compute estimate $\hat{\theta} = \hat{\theta}(X)$ which lies in Θ (or something close to Θ).
2. Point estimation of ftn of θ : compute estimate $\hat{\phi} = \hat{\phi}(X)$ of $\phi = g(\theta)$.
3. Interval (or set) estimation: compute set $C = C(X)$ in Θ which we think will contain θ_0 .
4. Hypothesis testing: choose between $\theta_0 \in \Theta_0$ and $\theta_0 \notin \Theta_0$ where $\Theta_0 \subset \Theta$.
5. Prediction: guess value of an observable random variable Y whose distribution depends on θ_0 . Typically Y is the value of the variable X in a repetition of the experiment.

Several schools of statistical thinking. Main schools of thought summarized roughly as follows:

- **Neyman Pearson:** A statistical procedure is evaluated by its long run frequency performance. Imagine repeating the data collection exercise many times, independently. Quality of procedure measured by its average performance when true distribution of X values is P_{θ_0} .
- **Bayes:** Treat θ as random just like X . Compute conditional law of unknown quantities given knowns. In particular ask how procedure will work on the data we actually got – no averaging over data we might have got.
- **Likelihood:** Try to combine previous 2 by looking only at actual data while trying to avoid treating θ as random.

We use Neyman Pearson approach to evaluate quality of likelihood and other methods.