STAT 804: 2004-1

Assignment 5

1. Suppose X and Y are stationary independent processes with respective spectra f_X and f_Y. Compute the spectrum of $Z = aX + Y$.

2. Suppose X and Y are jointly stationary processes and we observe them at times $1, \ldots, T$. Define the sample cross covariance $\hat{C}_{XY}(k) = \sum (X_t - \bar{X})(Y_{t+k} - \bar{Y})/T$ where are terms with index larger than T are interpreted as 0. Show that the sample cross covariance can be computed from the discrete Fourier transforms via

$$\hat{C}_{XY}(m) = \sum_{k=0}^{T-1} \hat{X}(k)\overline{\hat{Y}(k)} \exp(2\pi i k m/T)/T$$

   (or figure out the correct formula).

3. Derive the frequency response function for the recursive filter

$$Y_t = aY_{t-1} + X_t$$

   and plot the modulus squared and argument of the result for $a = 0.8$ and $a = 0.1$.

4. Compute and plot estimates of the spectrum for the time series `fake` for varying degrees of smoothing and compare the result to the spectrum of your fitted ARIMA model.

5. Let ϵ_t be a Gaussian white noise process. Define

$$X_t = \epsilon_{t-2} + 4\epsilon_{t-1} + 6\epsilon_t + 4\epsilon_{t+1} + \epsilon_{t+2}.$$

   Compute and plot the spectrum of X.

6. For the filters A: $y_t = x_t - x_{t-12}$, B: $y_t = x_t - x_{t-1}$ and C defined by applying A then B determine the power transfer functions, plot them and interpret their effect on a spectrum. What is the effect of these filters on seasonal series? (Consider what the spectrum of a series with a strong seasonal effect is like.)