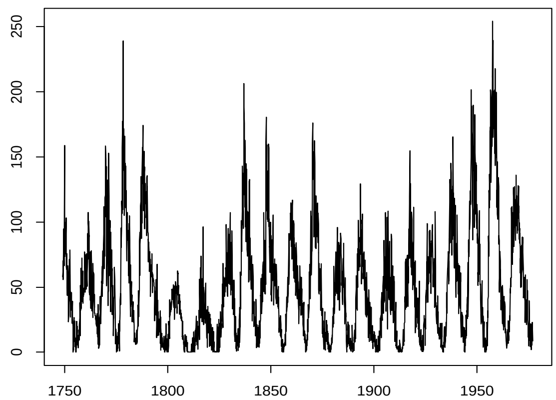
STAT 804: Lecture 1

Today:

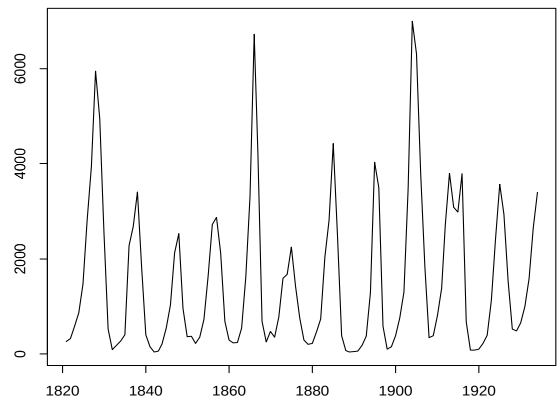
- Plots of some time series
- Discuss series using some of the jargon we will study.
- Basic classes of models.
- Existence of consistent estimates.

Plots of some series

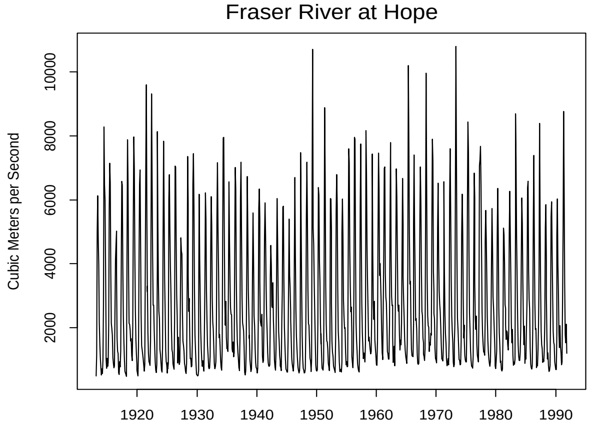
Mean Monthly Sunspot Numbers



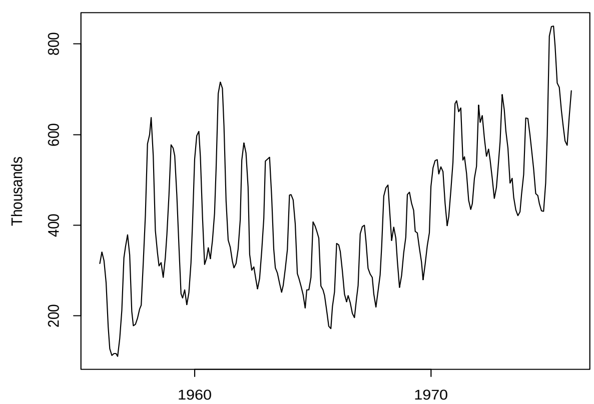
Annual Sales of Lynx
to Hudson's Bay Co.



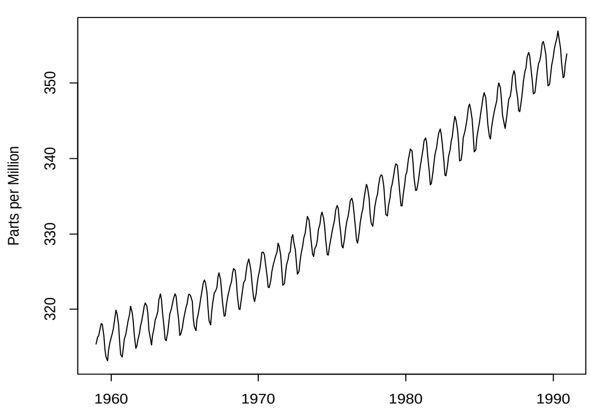
Mean Monthly Flow
Fraser River at Hope



Unemployment: Canada



CO2 concentration: Mauna Loa



Changes in length of day

Comments on the data sets:

- Top left: Sunspot data. Each month average number of sunspots is recorded. Note:

  - apparent periodicity

  - large variability when series at high level; small variability when at low level.

  This series is likely to be quite stationary over the time span we have been able to observe it, though it may have a nearly perfectly periodic component.

- Top right: Annual sales of lynx pelts to the Hudson's Bay Company. Note:

  - Clear cycle of about 10 years in length.

  - longer term cycle?

  - Is the cycle produced by a strictly periodic phenomenon or by a dynamic system close to a periodic system?

- Middle left: Mean monthly flow rates for the Fraser River at Hope. Note:
  - Signs of lower variability at low levels suggesting transformation.
  - Clear annual cycle which will have to be removed to look for stationary residuals.

- Middle right: Canadian monthly unemployment number. Note:
  - probable presence of slow upward trend; such a trend should be present in the presence of a growing population.
  - not stationary.
  - trend not too linear with some apparent long term cycles perhaps which produce an S shaped curve.

- Lower left: Carbon Dioxide above Mauna Loa (a Hawaiian volcano). Note:
  - Clear trend and an annual cycle
  - but you might well hope that after compensating for these the remainder would be stationary.

- Lower right: Changes in the length of the Earth's day. Note:
  - very smooth graph with long runs going up and down
  - suggests integration.

  We will look at differencing as a method of producing a series with less long range dependence.

Plots made with S-Plus using the following code:

```
postscript("tsplots.ps",horizontal=F)
par(mfrow=c(3,2))
tsplot(sunspots,main="Mean Monthly Sunspot Numbers")
tsplot(lynx,
   main="Annual Sales of Lynx\n to Hudson's Bay Co.")
tsplot(flow, ylab="Cubic Meters per Second",
   main="Mean Monthly Flow\nFraser River at Hope")
tsplot(unemployment,
   main="Unemployment: Canada",ylab="Thousands")
tsplot(co2, main="CO2 concentration: Mauna Loa",
   ylab="Parts per Million")
tsplot(changes,
   main="Changes in length of day", ylab="Seconds?")
dev.off()
```

Basic jargon

Defn: Stochastic process — family $\{X_i; i \in I\}$ of random variables indexed by a set I.

In practice the jargon is used only when the X_i are *not* independent.

If $I \subset$ Real Line, then we often call $\{X_i; i \in I\}$ a time series. Of course the usual situation is that i actually indexes a time point at which some measurement was made.

Two important special cases are I an interval in R, the real line, in which case we say X is a series in continuous time, and where $I \subset \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ in which case X is in discrete time.

List of some models used for time series:

- Stochastic Process Models (note the conflict of jargon).
  - Population models
    - Birth and Death Processes — which describe the size of a population in terms of random births and deaths.
    - Markov chain models — where the future depends on the present and not, in addition, on the past. Birth and Death processes are special cases.
    - Galton-Watson-Bienaymé process: a Markov chain model for the size of generations of a populations.

      Model specifies: size of nth generation is sum of iid family sizes of individuals in $n-1$st generation.

    * Branching processes: continuous time version of Galton-Watson-Bienaymé.

  - Diffusion models

    * Brownian Motion

    * Random Walk

    * Stochastic Diff'l Equations: models like

$$dX_t = \mu(X_t)dt + \sigma(X_t)dB_t$$

      where B is a Brownian motion.

- Linear Time Series Models — linear filters applied to white noise.

This course: about discrete time version of these linear time series models.

Assume throughout that we have data

$$X_0, X_1, \ldots, X_{T-1}$$

where the X_t are real random variables.

A **model** is a family $\{P_\theta; \theta \in \Theta\}$ of possible joint distributions for $\{X_0, \ldots, X_{T-1}\}$.

Goal: guess the true value of θ. (Notice that it is an assumption that the distribution of the data is, in fact one of the possibilities.)

The question is this: is it possible to guess the true value of θ?

Will collecting more data (increasing T) make more accurate estimation of θ possible?

The answer is no, in general.

Example: Galton-Watson-Bienaymé process – even with infinitely many generations you don't get enough data to nail down parameters.

Example: Suppose $(X_0, \ldots, X_{T-1})'$ has multivariate normal distribution with mean vector $(\mu_0, \ldots, \mu_{T-1})'$ and $T \times T$ variance covariance matrix $\mathbf{\Sigma}$.

Big problem: T data points but $T + T(T-1)/2$ parameters to estimate; this is not possible.

To make progress: put restrictions on parameters μ and $\mathbf{\Sigma}$.

For instance you might assume one of the following:

1. Constant mean: $\mu_t \equiv \mu$.

2. Linear trend; $\mu_t = \alpha + \beta t$.

3. Linear trend and sinusoidal variation:
$$\mu_t = \alpha + \beta t + \gamma_1 \sin\left(\frac{2\pi t}{12}\right) + \gamma_2 \cos\left(\frac{2\pi t}{12}\right)$$

Can estimate parameters by regression but still have problem: can't get standard errors.

For instance, we might estimate μ in 1) above using $\bar{X}$.

In that case

$$\begin{aligned}\text{Var}(\bar{X}) &= T^{-2}\text{Var}(\sum X_t) \\ &= T^{-2}\mathbf{1}^t\boldsymbol{\Sigma}\mathbf{1} \\ &= T^{-2}\sum_{s,t}\boldsymbol{\Sigma}_{st}\end{aligned}$$

where $\mathbf{1}$ is a column vector of T 1s.

So: we must model $\boldsymbol{\Sigma}$ as well as μ.

The assumption we will make in this course is of stationarity:

$$\begin{aligned}\text{Cov}(X_t, X_s) &= \text{Cov}(X_{t+1}, X_{s+1}) \\ &= \text{Cov}(X_{t+2}, X_{s+2}) \cdots\end{aligned}$$

If so then for all t and h we find

$$\begin{aligned}\text{Cov}(X_t, X_{t+h}) &= \text{Cov}(X_0, X_h) \\ &\equiv C_X(h)\end{aligned}$$

Call C_X autocovariance function of X.

Notice: $\mathbf{\Sigma}$ has

$C(0)$ down the diagonal

$C(1)$ down the first sub and super diagonals

$C(2)$ down the next sub and super diagonals and so on.

Such a matrix is called a Toeplitz matrix.