Some example plots

Plots of 5 series, fitted ACFs and fitted PACFs. The series are:

1. A generated $MA(2)$ series:

$$X_t = \epsilon_t + 0.8\epsilon_{t-1} - 0.9\epsilon_{t-2}$$

2. A generated $AR(2)$ series

$$X_t = \epsilon_t + X_{t-1} - 0.99X_{t-2}$$

3. A generated $AR(3)$ series:

$$X_t = \epsilon_t + 0.8X_{t-1} - X_{t-2}/3 + .8X_{t-3}/\sqrt{3}$$

4. Monthly sunspot counts over a 200 year period.

5. Annual rain fall measurements in New York City.

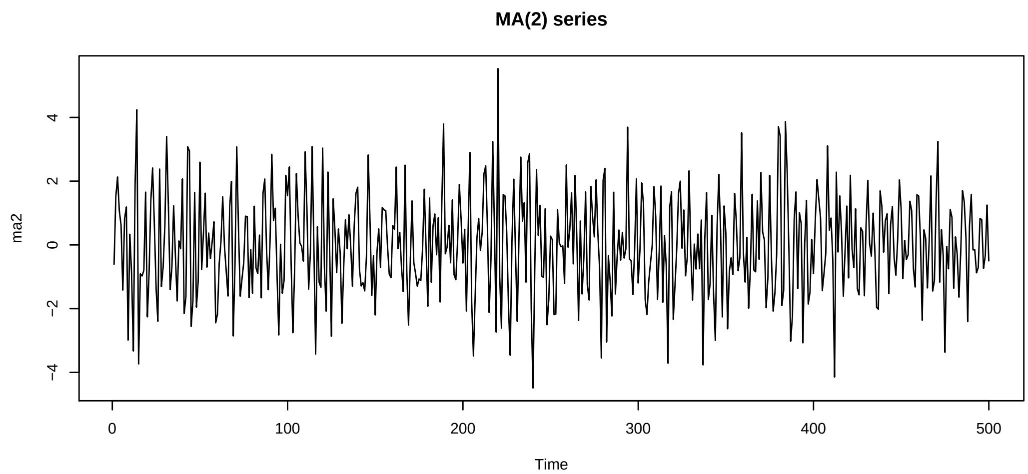
Here is the SPlus code I used to make the following plots. You should note the use of the function `acf` which calculates and plots ACF and PACF.

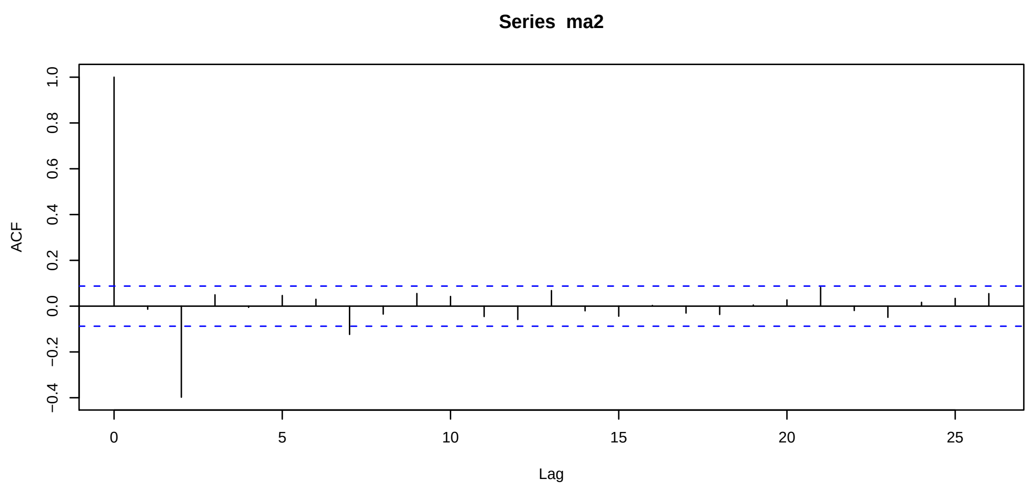
```
#
# Comments begin with #
#
# Begin by generating the series.
# AR series generated are NOT stationary.
# But: generate 10000 values from
# recurrence relation; use last 500.
# Asymptotic stationarity mean
# last 500 are pretty stationary.
#
n<- 10000
ep <- rnorm(n)
```

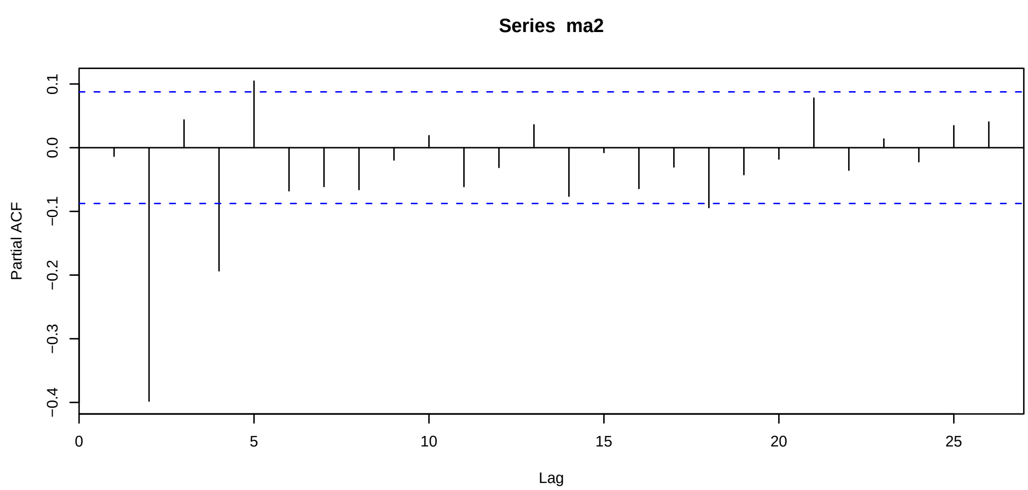
```
ma2 <-ep[3:502]+0.8*ep[2:501]-0.9*ep[1:500]
ar2 <- rep(0,n)
ar2[1:2] <- ep[1:2]
for(i in 3:n) {ar2[i] <- ep[i]
              + ar2[i-1] -0.99*ar2[i-2]}
ar2 <- ar2[(n-499):n]
ar2 <- ts(ar2)
ar3 <- rep(0,n)
ar3[1:3] <- ep[1:3]
for(i in 4:n){ar3[i] <- ep[i] + 0.8*ar3[i-1]
          -ar3[i-2]/3 +(0.8/1.712)*ar3[i-3]}
ar3 <- ar3[(n-499):n]
ar3 <- ts(ar3)
#
# The next line turns on a graphics device
#    -- in this case the graph will be
# made in postscript in file called ma2.ps.
# It will come out in portrait,
#   not landscape, format.
#
postscript(file="ma2.ps",horizontal=F)
```

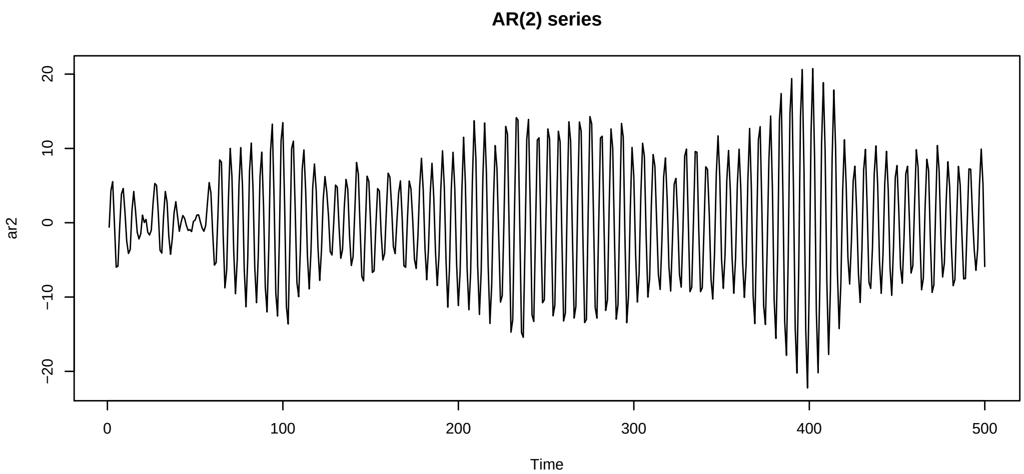
```
#
# The next line says to put 3 pictures
# in a single column on the plot
#
par(mfcol=c(3,1))
tsplot(ma2,main="MA(2) series")
acf(ma2)
acf(ma2,type="partial")
#
# When you finish a picture you turn
# off the graphics device
# with the next line.
#
dev.off()
#
#
#
postscript(file="ar2.ps",horizontal=F)
par(mfcol=c(3,1))
tsplot(ar2,main="AR(2) series")
acf(ar2)
acf(ar2,type="partial")
dev.off()
```

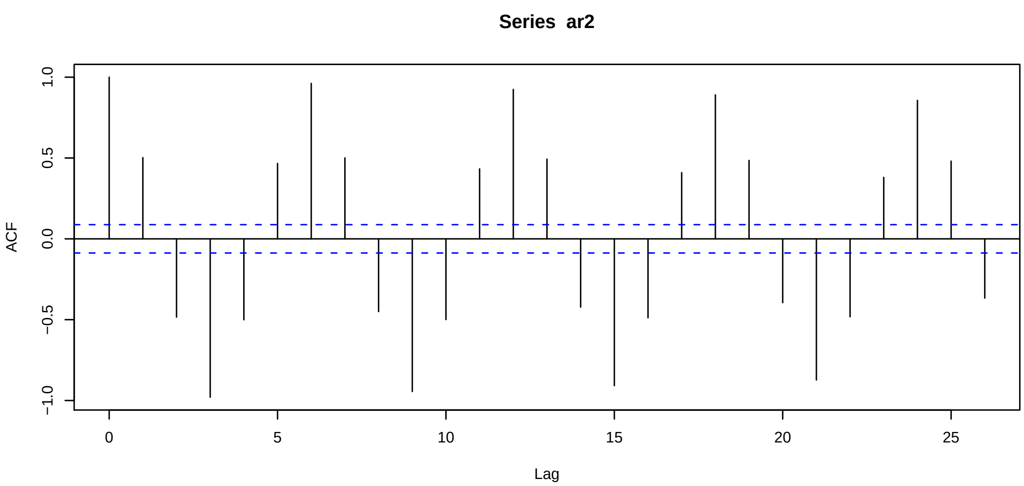
```
#
#
postscript(file="ar3.ps",horizontal=F)
par(mfcol=c(3,1))
tsplot(ar3,main="AR(3) series")
acf(ar3)
acf(ar3,type="partial")
dev.off()
#
#
postscript(file="sunspots.ps",horizontal=F)
par(mfcol=c(3,1))
tsplot(sunspots,main="Sunspots series")
acf(sunspots,lag.max=480)
acf(sunspots,lag.max=480,type="partial")
dev.off()
#
#
postscript(file="rain.nyc1.ps",horizontal=F)
par(mfcol=c(3,1))
tsplot(rain.nyc1,
            main="New York Rain Series")
acf(rain.nyc1)
acf(rain.nyc1,type="partial")
dev.off()
```

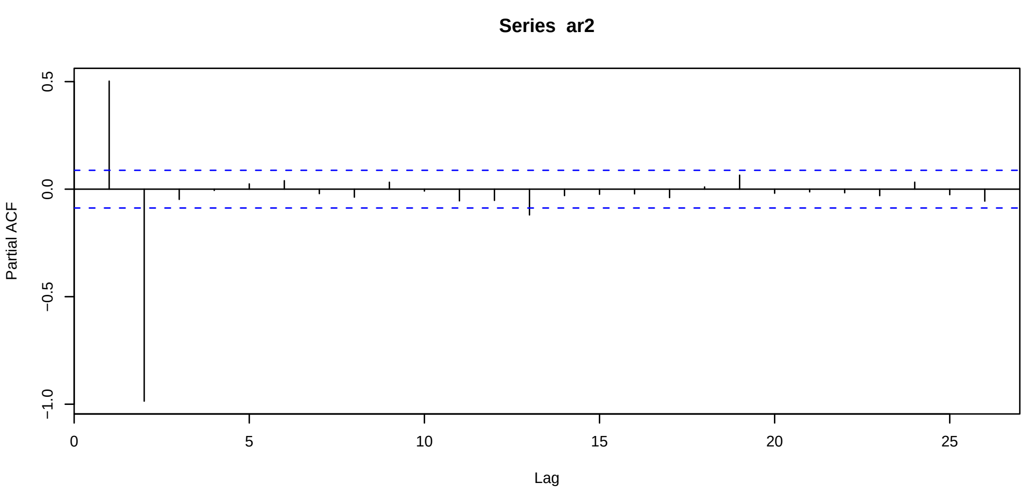

MA(2) series
ma2
Time
0
100
200
300
400
500
-4
-2
0
2
4

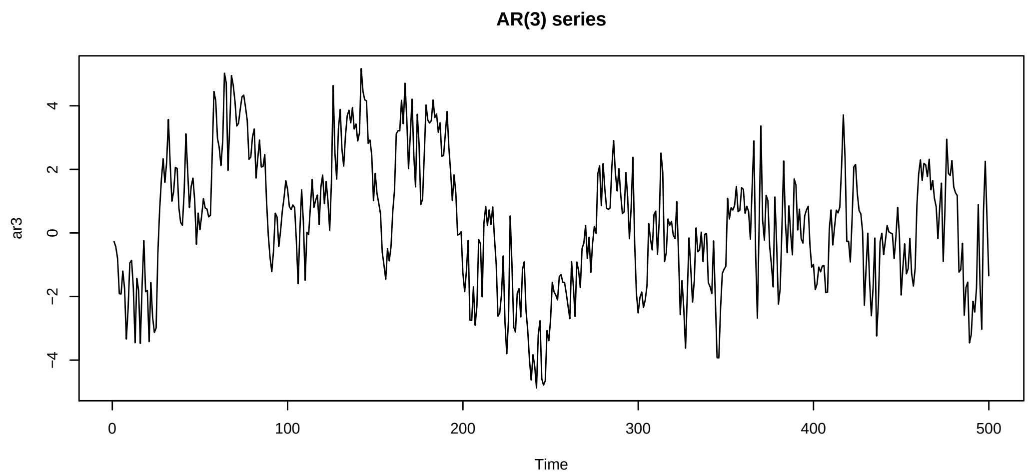

Series ma2
ACF
Lag
0
5
10
15
20
25
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0

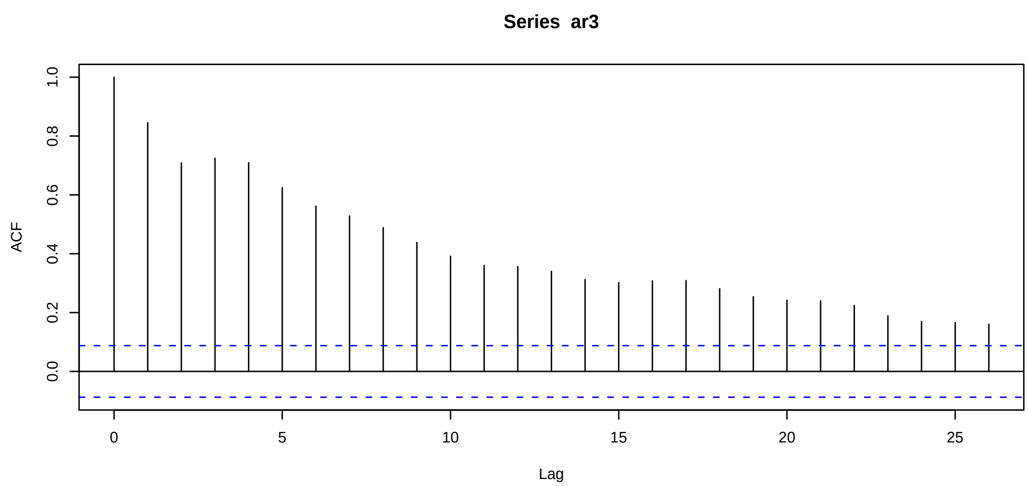

Series ma2
Partial ACF
Lag
0
5
10
15
20
25
-0.4
-0.3
-0.2
-0.1
0.0
0.1

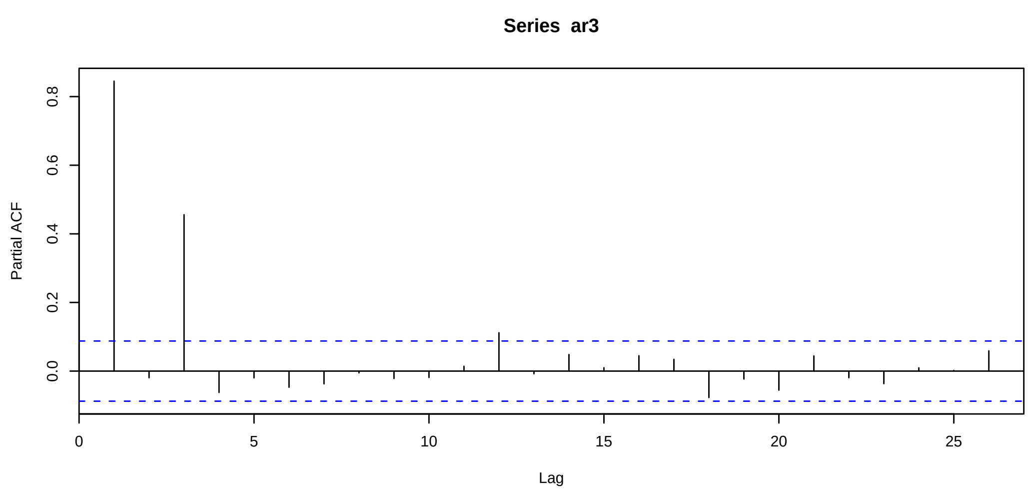

AR(2) series
ar2
Time
0
100
200
300
400
500
−20
−10
0
10
20


Series ar2
ACF
Lag
0
5
10
15
20
25
−1.0
−0.5
0.0
0.5
1.0

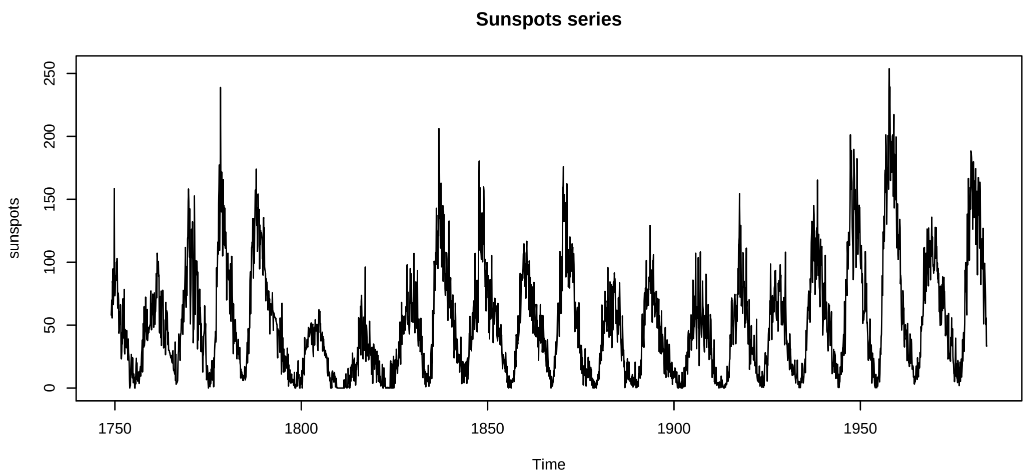

Series ar2
Partial ACF
Lag
0
5
10
15
20
25
−1.0
−0.5
0.0
0.5

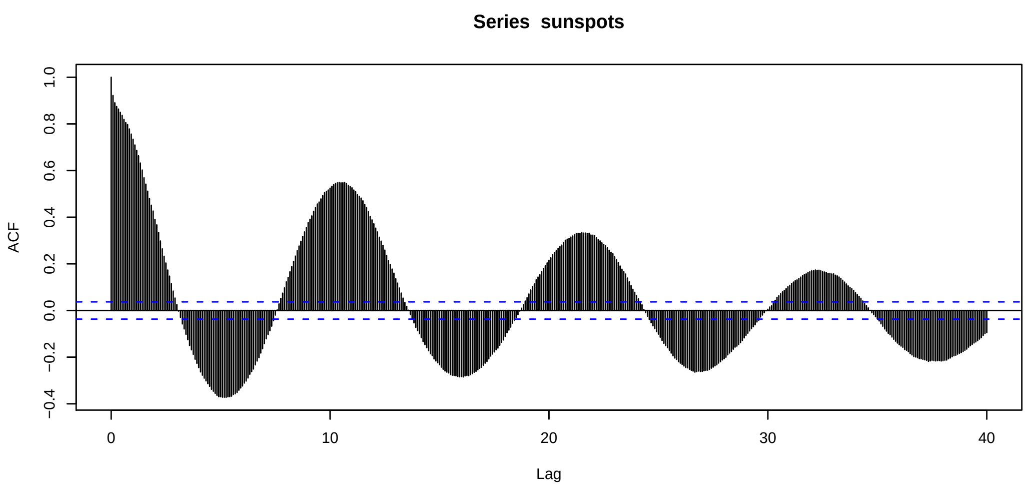

AR(3) series
ar3
Time
0
100
200
300
400
500

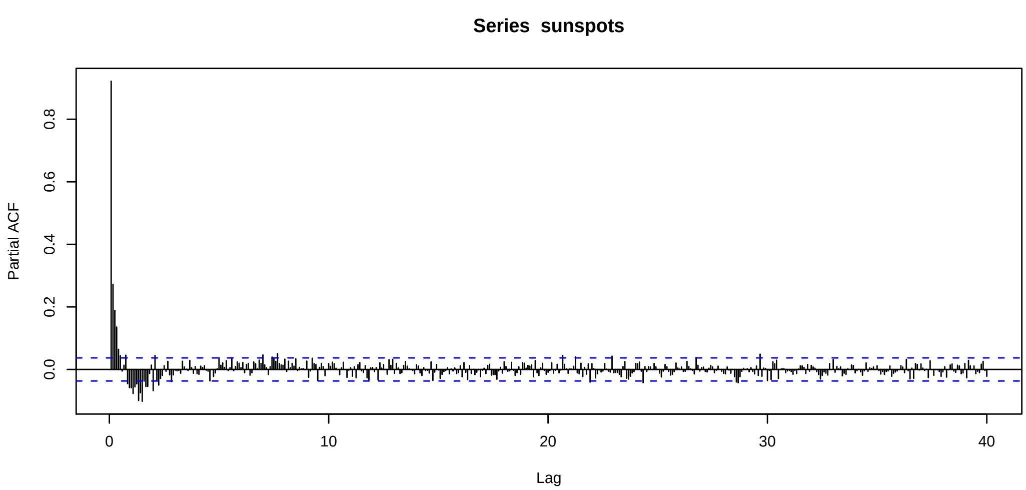

Series ar3
ACF
Lag


Series ar3
Partial ACF
Lag

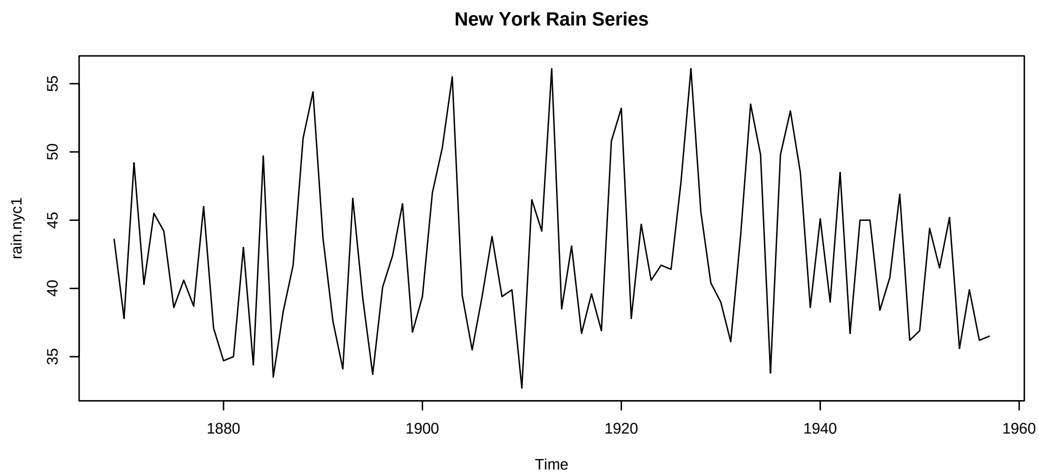
Sunspot data

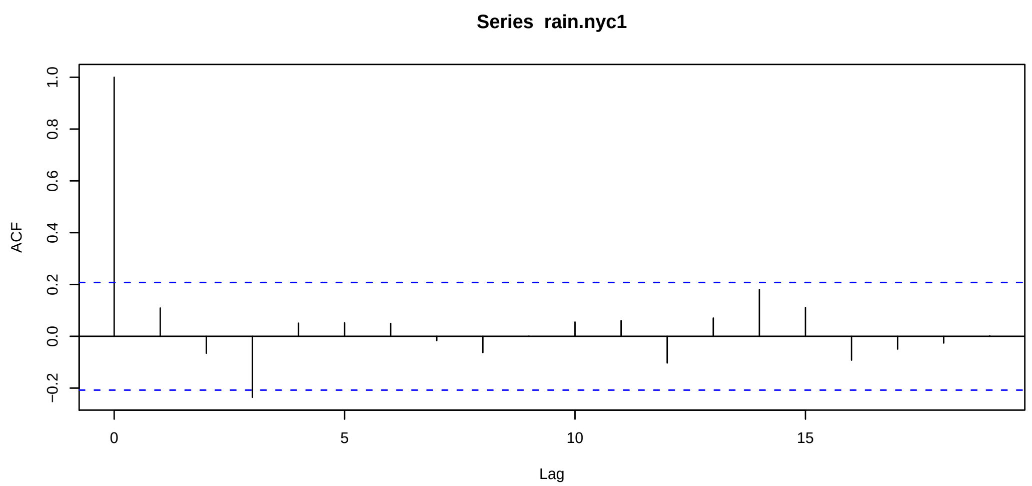


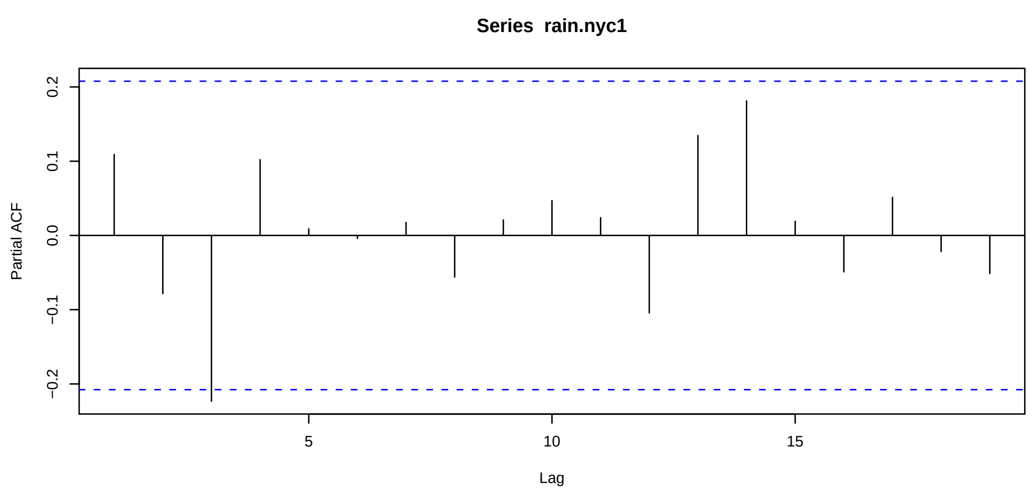




New York City Rainfall







You should notice:

- The MA series has an ACF which is near 0 for lags over 2 as predicted theoretically.

- The $AR(p)$ series have complicated ACF's but the PACFs vanish at lags more than p.

- The sunspot data has significant autocorrelation over many years but the PACF is pretty small after say 2 years or so. Notice that SPlus has labelled the lags in years so that the number 2 on the x axis corresponds to about 24 months. An $AR(p)$ model with p around 24 would then be a possibility.

- The New York rainfall series looks quite a bit like white noise.

Model identification summary

Simplest model identification tactic – look for pure MA or pure AR model. To do so:

1. compute sample autocorrelation function (ACF). Plot $\widehat{ACF}(h)$ against h. If the process is an $MA(q)$ then the ACF will be 0 after lag q.

2. compute sample partial ACF (PACF). Plot estimated $PACF(h)$ against h. If the process is an $AR(p)$ then the PACF will be 0 after lag p.