More than 1 process

Defn: Two processes X and Y are *jointly (strictly) stationary* if

$$\mathcal{L}(X_t, \ldots, X_{t+h}, Y_t, \ldots, Y_{t+h}) \\ = \mathcal{L}(X_0, \ldots, X_h, Y_0, \ldots, Y_h)$$

for all t and h.

Defn: X and Y are *jointly second order stationary* if each is second order stationary and also

$$C_{XY}(h) \equiv \text{Cov}(X_t, Y_{t+h}) = \text{Cov}(X_0, Y_h)$$

for all t and h.

Notice that negative values of h give, in general, different covariances than positive values of h.

Defn: If X is stationary the **autocovariance** function of X is $C_X(h) = \text{Cov}(X_0, X_h)$.

Defn: If X and Y are jointly stationary then the **cross-covariance** function is $C_{XY}(h) = \text{Cov}(X_0, Y_h)$.

Notice that $C_X(-h) = C_X(h)$ and $C_{XY}(h) = C_{YX}(-h)$ for all h and similarly for correlation

Defn: The **autocorrelation** function of X is

$$\rho_X(h) = C_X(h)/C_X(0) \equiv \text{Corr}(X_0, X_h).$$

the **cross-correlation** function of X and Y is

$$\begin{aligned}\rho_{XY}(h) &= \text{Corr}(X_0, Y_h) \\ &= C_{XY}(h)/\sqrt{C_X(0)C_Y(0)}.\end{aligned}$$

Fact: If X and Y are jointly stationary then $aX + bY$ is stationary for any constants a and b.