Seasonal Non-stationarity

Every winter the measured (not reported) unemployment rate in Canada rises.

Simple model with this feature: non-stationary mean of form

$$\mu_{t+S} = \mu_t$$

Typically $S = 12$ for monthly data or 4 for quarterly data (common in economic data).

Normally, $\mu_1, \ldots, \mu_S$ are not the same.

Defn: Deseasonalization is the process of transforming X to eliminate this sort of seasonal variation in the mean.

Method A: Regression. Estimate

$$\hat{\mu}_t = \frac{X_t + X_{t+S} + \cdots}{\# \text{ of terms}}$$

and then

$$\hat{Y}_t = X_t - \hat{\mu}_t$$

This is ordinary least squares regression with

$$\theta = \begin{bmatrix} \mu_0 \\ \vdots \\ \mu_{S-1} \end{bmatrix}$$

and

$$V = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}_{T \times S}$$

Method B: Seasonal differencing:

$$W_t = X_{t+S} - X_t$$

is stationary. BUT: if Y was an ARMA process then now W has a unit root.

Defn: A multiplicative ARIMA model written

$$ARIMA(p,d,q) \times (P,D,Q)_S$$

has the form:

$$\Phi(B^S)\phi(B)(I-B^S)^D(I-B)^d X = \Psi(B^S)\psi(B)\epsilon$$

where Φ, ϕ, Ψ, ψ polynomials of degree P, p, Q, q respectively with all roots outside the unit circle (and other technical conditions – see assignment 2, question 1) and ϵ is white noise.

As an example consider the model

$$(1-\Phi_1 B^{12})(1-\phi_1 B)X = (1-\Psi_1 B^{12})(1-\psi_1 B)\epsilon$$

which is an $ARIMA(1,0,1) \times (1,0,1)_{12}$ model.

Multiplying this out we get

$$
\begin{aligned}
(I - \phi_1 B - \Phi_1 B^{12} + \phi_1 \Phi_1 B^{13})X \\
= (I - \psi_1 B - \Psi_1 B^{12} + \psi_1 \Psi_1 B^{13})\epsilon
\end{aligned}
$$

which looks like an $ARMA(13, 13)$.

Key point: new model has only 4 parameters (plus the variance of ϵ) instead of the 26 for a general $ARMA(13, 13)$.