

Spectral analysis of sunspot data

I consider a model of the form

$$S_t = \mu + \sum_{j=1}^k \{a_j \cos(2\pi\omega_j t) + b_j \sin(2\pi\omega_j t)\} + W_t$$

where W is some $\text{ARMA}(p, q)$.

Spectrum has several peaks located using

```
bump <- function(xx,a,b, n=1000){  
  f <- seq(a, b, length = n)  
  x <- unlist(c(xx))  
  # To make the following code work must  
  # remove time series attributes of xx  
  nn <- 1:length(x)  
  args <- outer(f, nn, "*") * 2 * pi  
  cosines <- t(cos(t(args)) * x)  
  sines <- t(sin(t(args)) * x)  
  one <- rep(1, length(x))  
  per <- ((cosines %*% one)^2  
    + (sines %*% one)^2)/length(x)  
  top <- max(per)  
  f[per == top]  
}
```

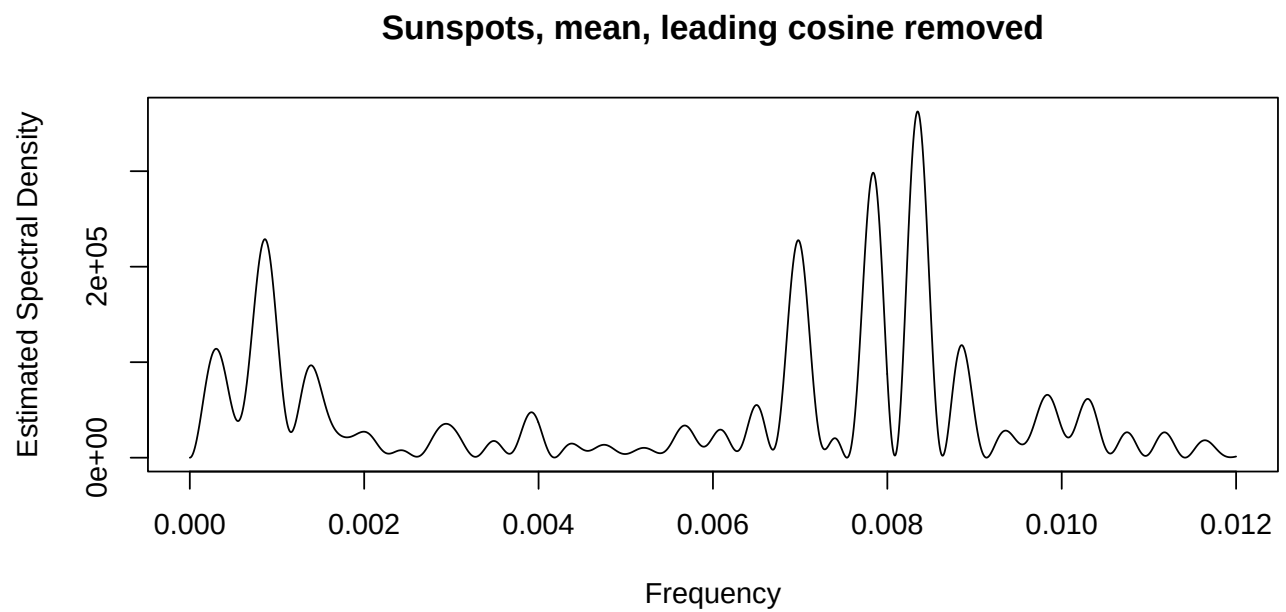
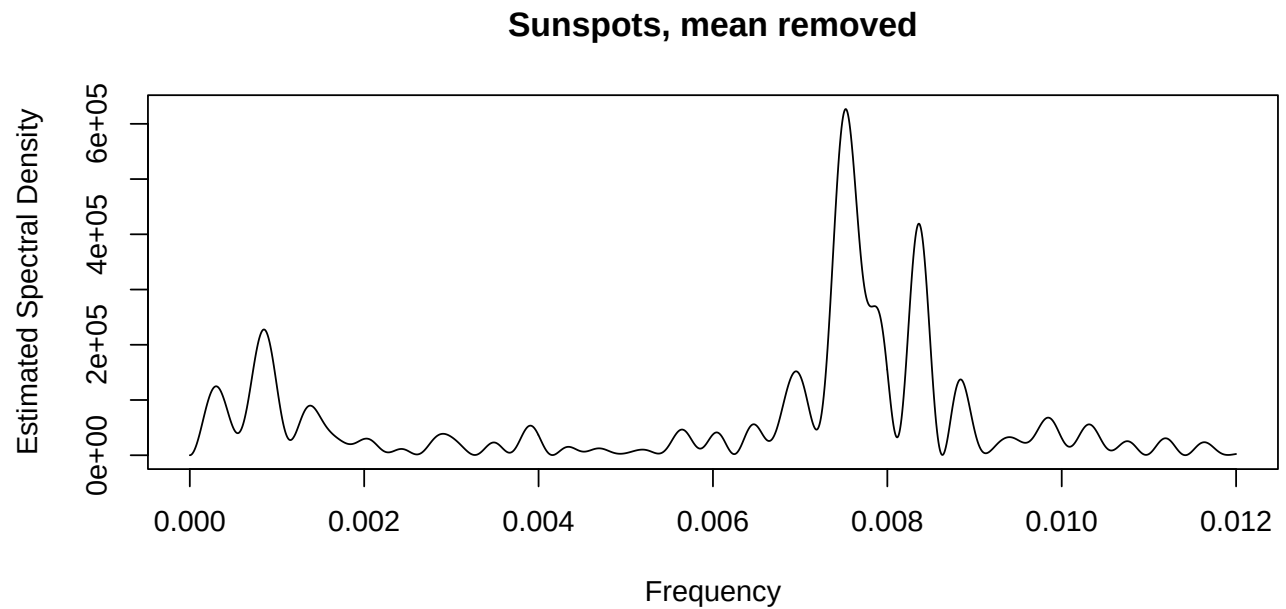
Output from R:

```
f <- bump(x,1/135,1/130,n=2000)

# > f
# [1] 0.007536817
f <- bump(x,0.00753, 0.00755,n=2000)
# > f
# [1] 0.007536783

n <- length(x)
xr <- cbind(cos(2*pi*(1:n)*f),sin(2*pi*(1:n)*f))
lm(x~xr)
#
# Call:
# lm(formula = x ~ xr)
#
# Coefficients:
# (Intercept)          xr1          xr2
#      50.98      27.67      10.57
#
xres <- residuals(.Last.value)
```

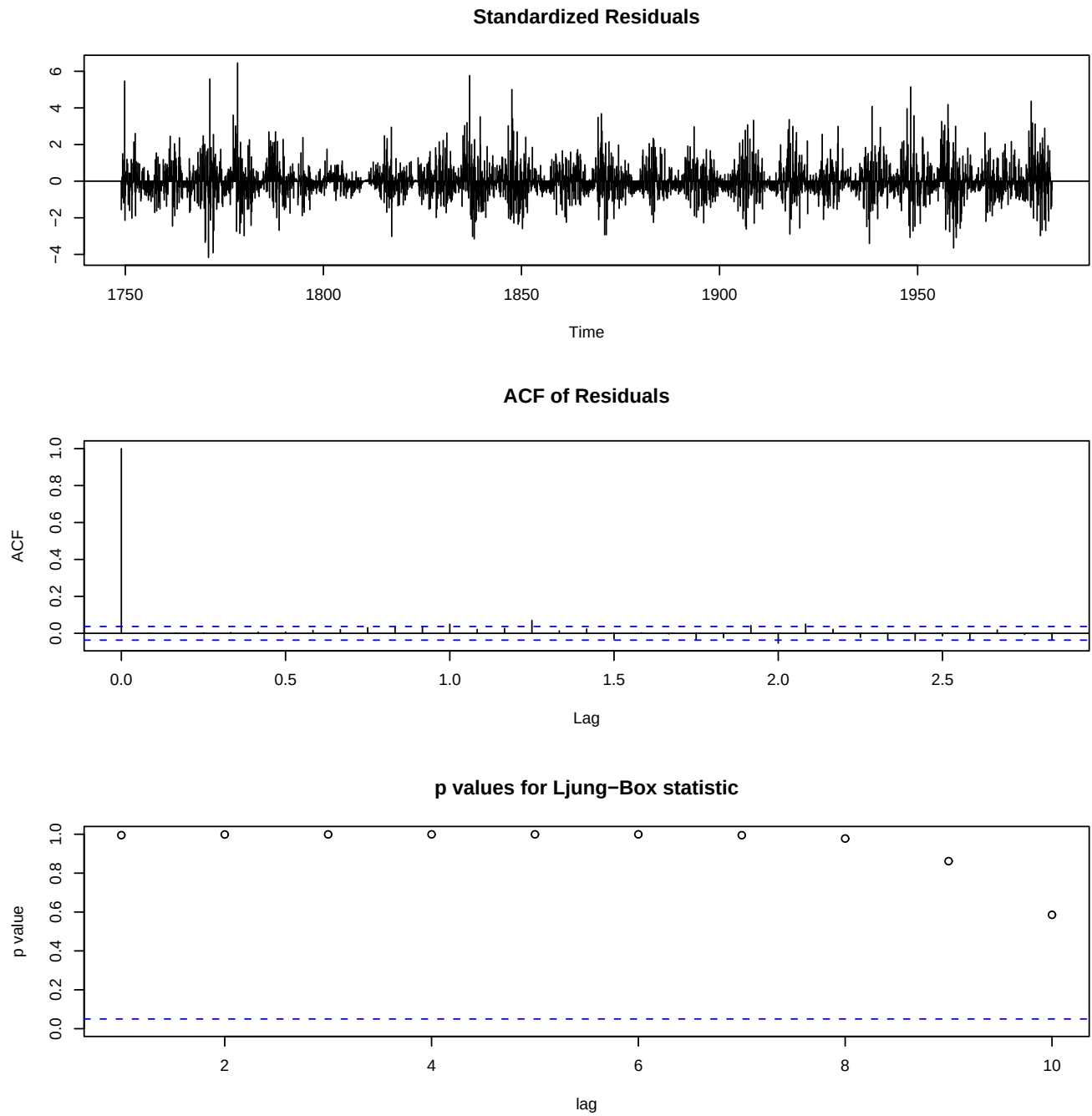
Compare spectra with and without cosine term.



One bump removed, one exposed. Remove 5 more

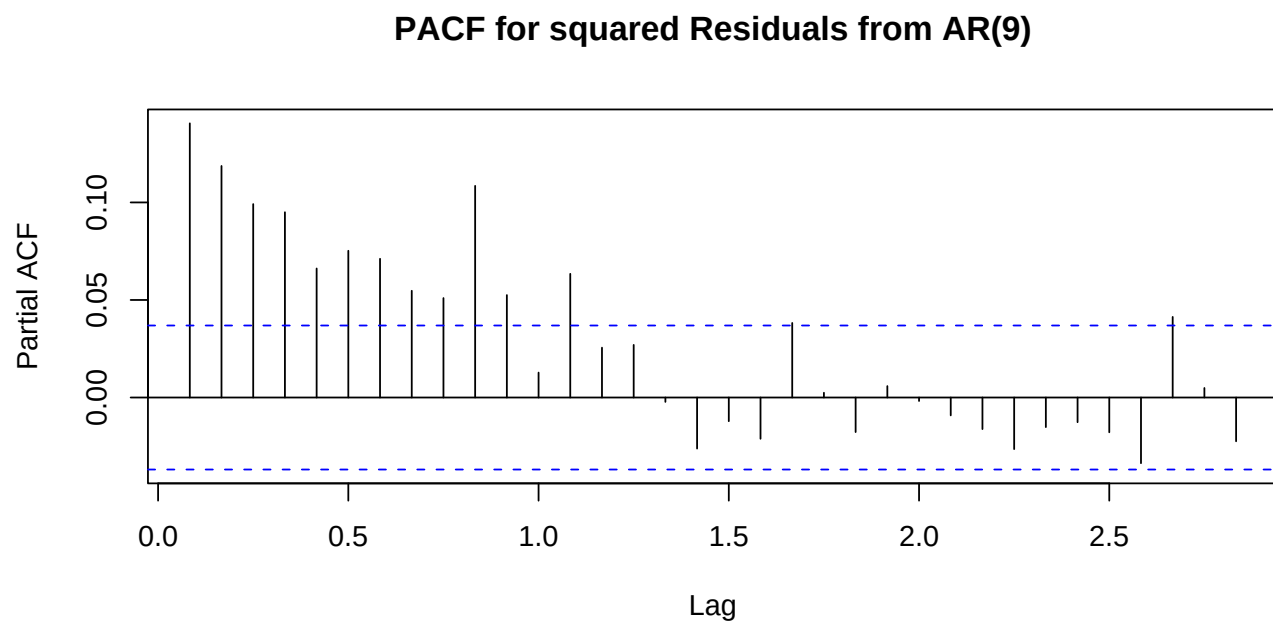
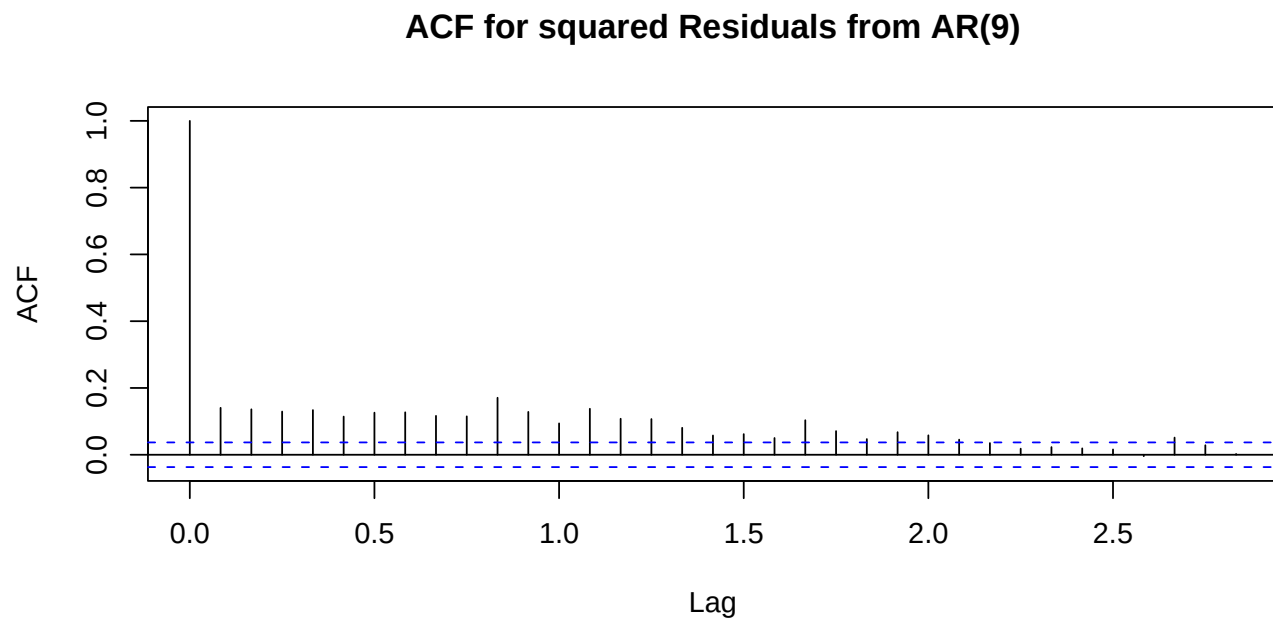
```
> f <- c(f,bump(xres,0.0077,0.008,n=2000))
> f <- c(f,bump(xres,0.008,0.009,n=2000))
> f <- c(f,bump(xres,0,0.0006,n=2000))
> f <- c(f,bump(xres,0.0006,0.0012,n=2000))
> f <- c(f,bump(xres,0.0012,0.0016,n=2000))
> round(1/(12*f),3)
[1] 11.057 10.632 9.982 275.162 96.738 59.913
> xra <- cbind(xr,cos(2*pi*(1:n)*f[2]),sin(2*pi*(1:n)*f[2]))
...
> xra <- cbind(xra,cos(2*pi*(1:n)*f[6]),sin(2*pi*(1:n)*f[6]))
> lm(x~xra)
> xres2 <- residuals(.Last.value)
> ar(xres2,method="mle")
order selected 9 sigma^2 estimated as 233.7
> arima.fit <- arima(sunspots,order=c(9,0,0),xreg=xra)
> arima.res <- residuals(arima.fit)
```

Diagnostic plot tsdiag:

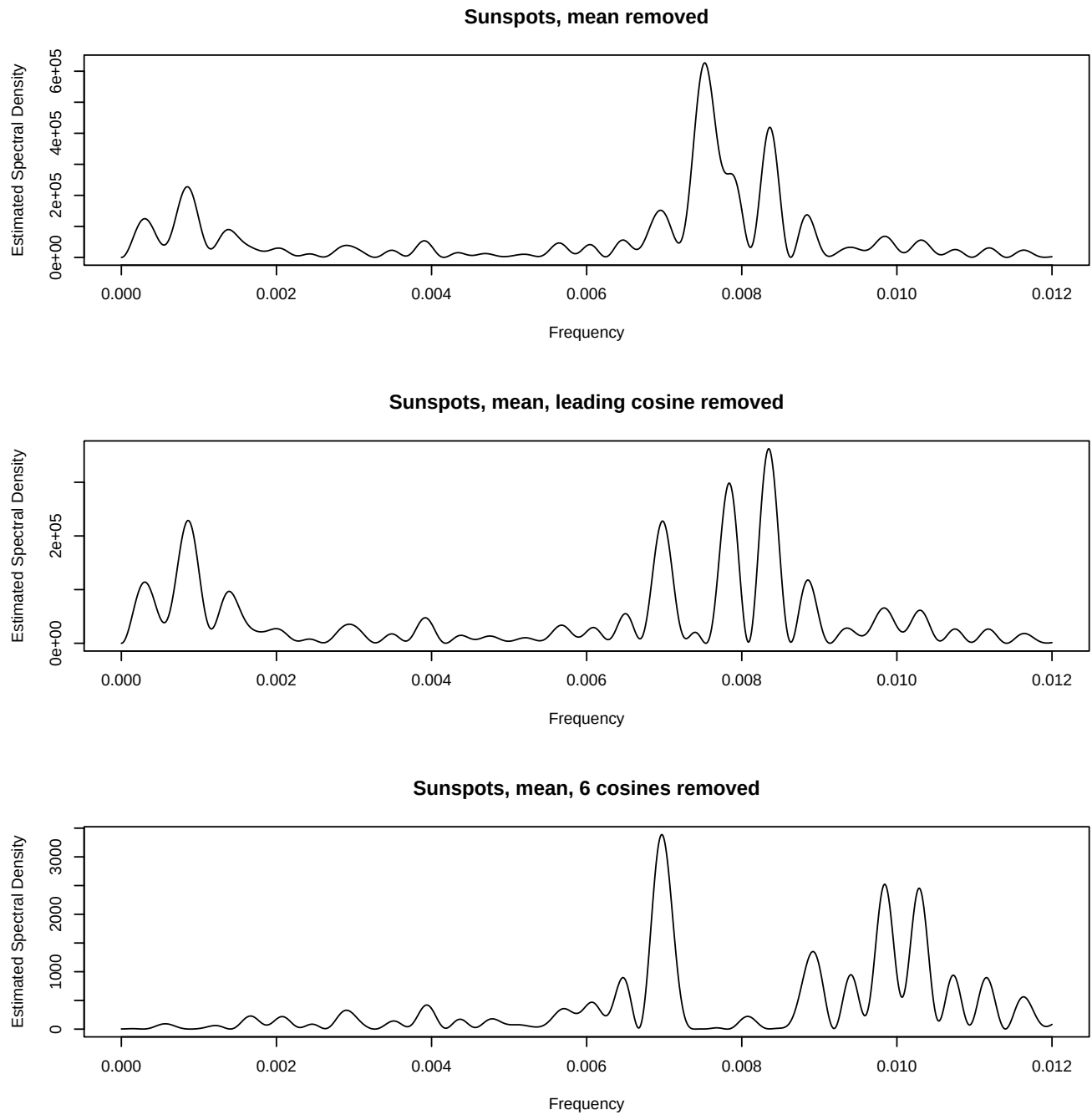


Notice waves: autocorrelated sizes?

Try ACF of Residuals squared



Plot all spectra together



All spectra together on same plot

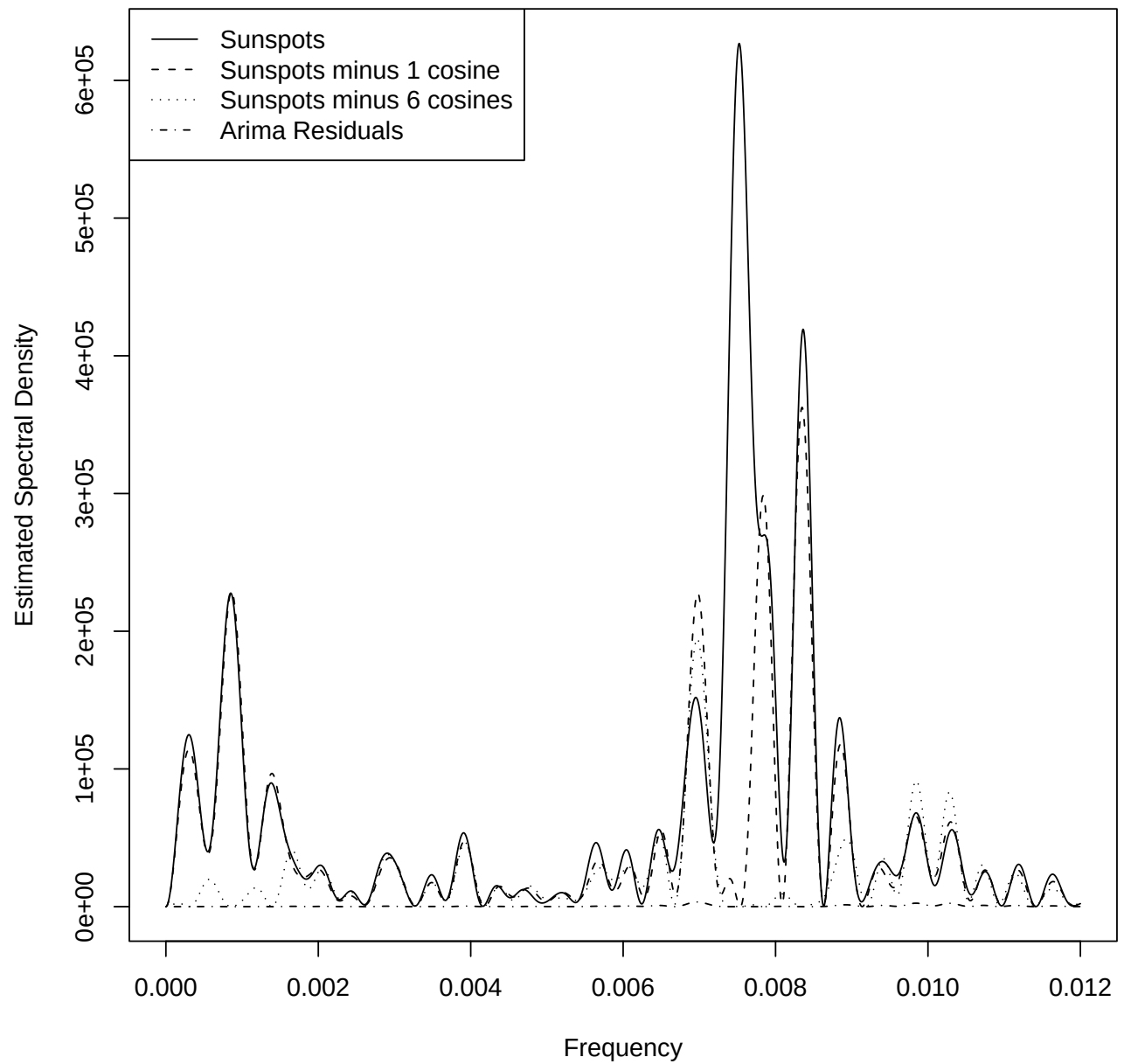


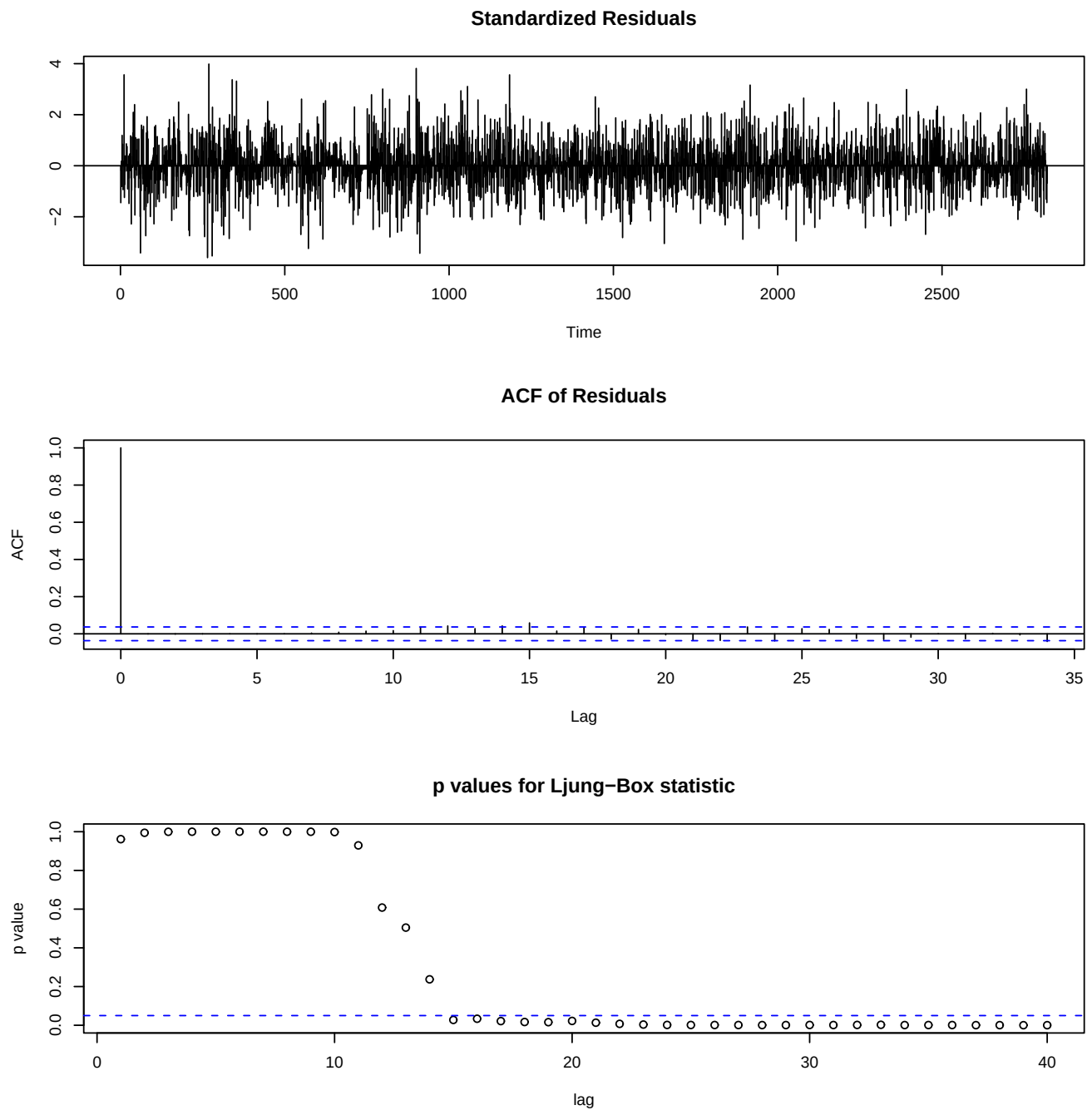
Table of estimates and SEs

Parameter	Estimate	SE
a_1	0.5299	0.0188
a_2	0.0937	0.0213
a_3	0.0743	0.0213
a_4	0.0800	0.0214
a_5	0.0309	0.0214
a_6	0.0441	0.0214
a_7	-0.0083	0.0213
a_8	0.0002	0.0213
a_9	0.0835	0.0188
σ	233.7	

Parameter	Estimate	SE	Period (yr)
μ	50.33	4.22	
β_1	18.36	2.80	9.982
γ_1	4.68	2.80	
β_2	1.76	2.95	10.632
γ_2	-16.27	2.96	
β_3	27.27	2.99	11.057
γ_3	9.85	2.99	
β_4	-8.96	5.46	59.913
γ_4	-5.17	5.58	
β_5	8.81	5.78	96.738
γ_5	12.31	6.03	
β_6	4.71	6.34	275.162
γ_6	-7.78	5.58	

Repeat calculation on square root of sunspots.

Fitted model is AR(11).



Plot of fitted values superimposed on square root of sunspot numbers.

