STAT 830

Problems: Assignment 2

1. Suppose X and Y have joint density $f(x, y)$. Prove from the definition of density *given in class* that the density of X is $g(x) = \int f(x, y) \, dy$.

2. Suppose X is Poisson(θ). After observing X a coin landing Heads with probability p is tossed X times. Let Y be the number of Heads and Z be the number of Tails. Find the joint and marginal distributions of Y and Z.

3. Let p_1 be the bivariate normal **density** with mean 0, unit variances and correlation ρ and let p_2 be the standard bivariate normal **density**. Let $p = (p_1 + p_2)/2$.

   (a) Show that p has normal margins but is not bivariate normal.

   (b) Generalize the construction to show that there rv's X and Y such that X and Y are each standard normal, X and Y are uncorrelated but X and Y are not independent.

   (c) In `R` make a contour plot of p for $\rho = 0.7$.

4. Suppose X and Y are independent with $X \sim N(\mu, \sigma^2)$ and $Y \sim N(\gamma, \tau^2)$. Let $Z = X + Y$. Find the distribution of Z given X and that of X given Z.

5. Compute the characteristic function, cumulants and the first 5 central moments for the Gamma distribution with shape parameter α and scale parameter β. You may feel free to use MAPLE to help take derivatives or whatever.

6. Suppose X has the Beta(α, β) density

$$f(x; \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1 - x)^{\beta-1} 1(0 < x < 1)$$

Find the distribution of $Y = X/(1 - X)$.

7. If (X_1, X_2) has the density

$$f(x_1, x_2) = 24x_1x_21(0 < x_1)1(0 < x_2)1(x_1 + x_2 < 1)$$

what is the distribution of $X_1 + X_2$?

8. Suppose X and Y are iid $N(0, \sigma^2)$.

   (a) Show that $X^2 + Y^2$ and $X/(X^2 + Y^2)^{1/2}$ are independent.

   (b) Show that $\Theta = \arcsin(X/(X^2 + Y^2)^{1/2})$ is uniformly distributed on $(-\pi/2, \pi/2]$.

   (c) Show X/Y is a Cauchy random variable.

9. From the text p45 # 13.

10. From the text p45 # 16.

11. From the text: page 61 #18

12. From the text p61 # 21.

Due date: 28 September 2011 in class.