STAT 830

Problems: Assignment 3

NOTE: you only need to do problems 1 and 2 if you haven't already done them on assignment 2.

1. Suppose X and Y are independent with $X \sim N(\mu, \sigma^2)$ and $Y \sim N(\gamma, \tau^2)$. Let $Z = X + Y$. Find the distribution of Z given X and that of X given Z.

2. Suppose X and Y are iid $N(0, \sigma^2)$.

   (a) Show that $X^2 + Y^2$ and $X/(X^2 + Y^2)^{1/2}$ are independent.

   (b) Show that $\Theta = \arcsin(X/(X^2 + Y^2)^{1/2})$ is uniformly distributed on $(-\pi/2, \pi/2]$.

   (c) Show X/Y is a Cauchy random variable.

3. From the text on page 69, #7. I want you to plot the exact probability, and the two approximations mentioned (Mill's inequality and Chebyshev's inequality) on one graph from $t = 1$ to $t = 3$. Then please plot the logarithms of these approximations against t from $t = 2$ to $t = 5$.

4. From the text page 82 # 1.

5. From the text page 82 # 2.

6. From the text page 83 # 4.

7. From the text page 83 # 5.

8. From the text page 84 # 14.

9. From the text page 84 # 15.

10. If $(X_1, Y_1), \ldots, (X_n, Y_n)$ are independent bivariate normal random variables find the limiting distribution of $n^{1/2}(r - \rho)$ where r is the sample correlation coefficient and ρ is the population correlation. HINTS: the problem is easier for $\mu_X = \mu_Y = 0$ and $\sigma_X = \sigma_Y = 1$ so prove that you can assume these values for the parameters without loss of generality. Next remember that the correlation coefficient can be computed

from the 5 summary statistics $\bar{X}, \bar{Y}, \bar{X^2}, \bar{Y^2}, \bar{XY}$. Use the central limit theorem (multivariate version) to compute an approximate normal distribution for this vector.

Due date: 17 October 2011 in class.