Lecture 13: Method of Moments

- To estimate p parameters solve

$$\bar{X} = \mathrm{E}_\theta(X)$$
$$\bar{X^2} = \mathrm{E}_\theta(X^2)$$
$$\vdots \quad \vdots$$
$$\bar{X^p} = \mathrm{E}_\theta(X^p)$$

- Essentially always consistent.
- Did Gamma(α, β) example.
- Used as starting points for Newton-Raphson.
- To solve $g(\theta) = 0$ begin with initial value θ_0 and iteratively define

$$\theta^{(k+1)} = \theta^{(k)} - \left(Dg(\theta^{(k)})\right)^{-1} g(\theta^{(k)}).$$

Here Dg is the $p \times p$ matrix with i,jth

$$\frac{\partial g_i(\theta)}{\partial \theta_j}.$$

Lecture 13: quadratic approximation to ℓ

- Two term Taylor expansion of ℓ about $\hat\theta$:

$$2\left[\ell(\hat\theta)-\ell(\theta)\right] \approx (\theta-\hat\theta)^T\mathcal{I}(\theta_0)(\theta-\hat\theta)$$

  where minus the second derivative matrix has been replaced by the Fisher information matrix.
- Worked on large sample theory of above evaluated at $\theta=\theta_0$.
- Began with $X \sim MVN_p(0,\Sigma)$ and showed

$$X^T\Sigma^{-1}X \sim \chi^2_p.$$

- Then started general theory of quadratic form:

$$X^tBX = (AZ)^TB(AZ) = Z^TQZ$$

  where $Q = A^TBA$ and $AA^T=\Sigma$.
- Write $Q = P\Lambda P^T$ where P is orthogonal ($PP^T = P^TP = I$) and Λ is diagonal. Columns of P are eigenvectors of Q and diagonal entries of Λ are eigenvalues.

Coverage in the text

- Method of moments in Chapter 9.
- See “course notes” on web pages 130-131.