Lecture 18: The delta method

- If f is smooth from $\mathbb{R}^p$ to $\mathbb{R}^q$ and $\sqrt{n}(X_n - x)$ converges in distribution to $MVN_p(0, \Sigma)$ then
$$\sqrt{n}(f(X_n) - f(x)) \Longrightarrow MVNN_q(0, D\Sigma D^T)$$
where D is the $q \times p$ matrix with i, j entry
$$\frac{\partial f_i}{\partial x_j}$$
evaluated at x.
- I did an example: $X_1, \ldots, X_n$ iid with mean μ, SD σ, and higher moments $\mu_k = \mathrm{E}\left[(X-\mu)^k\right]$.
- We studied *Coefficient of Variation* (CV) given by
$$\sigma/\mu$$
which is estimated by
$$\frac{\sqrt{\bar{X^2} - \bar{X}^2}}{\bar{X}}.$$
- Define $f(u, v) = \sqrt{v - u^2}/u$ to see that

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- The derivative matrix D is

$$\left[-\frac{1}{\sigma} - \frac{CV}{\mu} \quad \frac{1}{2\mu\sigma} \right]$$

- I took the time to work out the variance covariance of

$$[X \quad X^2]^T$$

to get the matrix

$$\Sigma = \begin{bmatrix} \sigma^2 & \mu_3 + 2\mu\sigma^2 \\ \mu_3 + 2\mu\sigma^2 & \mu_4 - \sigma^4 + 4\mu\mu_3 + 4\mu^2\sigma^2 \end{bmatrix}$$

- After algebra

$$\tau^2 = CV^4 + \frac{\mu_4 - \sigma^4}{\sigma^4} \times \frac{CV^2}{4} + \frac{\mu_3}{\mu^3}$$

which is different by a sign from class.

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- I talked about profiling for LD50.
- I talked about using $\hat{\tau}$ for confidence intervals (and hypothesis tests.
- I talked about the duality between confidence sets and families of hypothesis tests.

Coverage in the text

- Chapter 9.9, pp 131–133.
- Course slides “Convergence in Distribution”: 21-28.
- Also red slides 29-31.
- See “course notes” on web pages 68-71.