Lecture 6: Statistical Functionals

- A statistical functional is a parameter which has a meaning for all or at least most cdfs F.
- Techniclly a statistical functional is a map $F \to T(F)$ which associates a cdf with a real number.
- Classic examples are mean, median, quantiles, other moments, characteristic functions, moment generating functions.
- Less obvious examples are things like

$$T(F) = \mathrm{median}(\bar{X}_n)$$

  the median of the distribution of $\bar{X}$ for a sample of size n from F.
- A linear functional has

$$T(aF + bG) = aT(F) + bT(G).$$

- Example linear functionals are

$$T_r(F) = \int r(x)dF(x) = \int r(x)f(dx) = \mathrm{E}_F(r(X)).$$

Bootstrap and plug-in estimation

- The plug-in estimate of $T(F)$ is $T(\hat{F}_n)$.
- So the plug-in estimate of $T_r(F)$ is

$$\int r(x) d\hat{F}_n(x) = \frac{1}{n} \sum r(X_i).$$

- Notice how to do the integral: sum of values times probabilities because $\hat{F}_n$ is a discrete distribution with n possible values and probability $1/n$ for each of those values.
- To compute more complicated plug in estimates we simulate.
- This is called *bootstrapping*.
- Generate samples of size n from $\hat{F}_n$ and evaluate the object being studied for each sample. Average to get a Monte Carlo estimate of the $\hat{F}$ expected value of the object.
- I bootstrapped the t pivot and $\bar{X} - \mu$.
- Then I studied the quality of resulting confidence intervals by Monte Carlo!

Course Coverage

- Chapter 7.2
- Chapter 8.1-3.