Lecture 9: Likelihood Asymptotics

- I stated Jensen's inequality
- I proved that if θ is identifiable then

$$\frac{\ell(\theta_0) - \ell\theta}{n} \to \mu(\theta) \geq 0$$

  with a strict inequality except at $\theta = \theta_0$.
- The convergence is uniform on some closed interval around θ_0.
- So if $\hat\theta_n$ maximizes the likelihood over this interval then

$$\hat\theta_n \to \theta_0$$

  in probability; we say $\hat\theta_n$ is *consistent*.
- Moreover

$$\lim_{n\to\infty} P_{\theta_0}(U(\hat\theta_n) = 0) = 1.$$

- There is a consistent root of the likelihood equations.

The multivariate normal distribution

- We have
$$\frac{U_n(\theta_0)}{n} \Longrightarrow MNV(0, \Sigma)$$
where $\Sigma = \mathrm{Var}_{\theta_0}(U_1(\theta_0))$.
- I reviewed the definition of the multivariate normal distribution.
- $Z \sim N(0,1)$ means $Z \in \mathbb{R}^1$ has density
$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}.$$
- $Z = [Z_1, \ldots, Z_p]^T \in \mathbb{R}^p$ has $MVN_p(0, I)$ distribution if $Z_1, \ldots, Z_p$ are iid N(0,1).

The multivariate normal distribution

- $X \in \mathbb{R}^p$ is multivariate normal if it has the same distribution as $AZ + b$ for $Z \sim MVN_q(0, I)$, A a $p \times q$ matrix of consants and $b \in \mathbb{R}^p$.
- Then $\mathrm{E}(X) = b \equiv \mu$ and $\mathrm{Var}(X) = AA^T$.
- Use moment generating functions to see that $BB^T = AA^T$ implies $AZ + \mu$ and $BZ + \mu$ have the same distribution denoted $MVN(\mu, \Sigma)$ where $\Sigma = bB^T = AA^T$.
- Density of X was given for Σ non-singular.
- Example of singular Σ given using vector of fitted residuals in multiple regression.

Course coverage

- Chapter 9.3-7.
- Chapter 2.10.