STAT 801=830

Problems: Assignment 2

1. Consider the empirical distribution funct $\hat{F}_n(x)$ for a sample $X_1, \ldots, X_n$ from a cdf F. In this problem I want you to compare several confidence limits for $F(x)$:

   - The pointwise interval based on the approximately normal pivot

   $$\frac{\sqrt{n}(\hat{F}_n(x) - F(x))}{\sqrt{\hat{F}_n(x)[1 - \hat{F}_n(x)]}}$$

   - The pointwise interval based on the approximately normal pivot

   $$\frac{\sqrt{n}(\hat{F}_n(x) - F(x))}{\sqrt{F(x)[1 - F(x)]}}$$

   - The simultaneous interval based on the Dvoretsky-Kiefer-Wolfowitz inequality as described in the text.
   - The simultaneous interval based on the assertion that

   $$\sup_x\{\sqrt{n}|\hat{F}_n(x) - F(x)|\} \xrightarrow{d} \sup_x\{|B_0(x)|\}$$

   where B_0 is a Brownian Bridge.

   I want you to do the following to make the comparison for 95% intervals:

   (a) Generate a sample of size 20 from the Uniform[0,1] distribution. Plot, on one graph, the 4 intervals above along with the true cdf F for x running from 0 to 1.

   (b) Generate 1000 samples of size 20 from the same distribution and for each x in $\{0.1, 0.2, \ldots, 0.9\}$ estimate the pointwise coverage probability for each procedure.

   (c) For the same samples estimate the simultaneous coverage probability for the set of 9 x values in the previous problem.

Then I want you to summarize in a paragraph the conclusions of the comparisons. In making the comparisons you need to know that

$$P\left(\sup_x\{|B_0(x)|\} \geq 1.358\right) = 0.05.$$

2. From the text Chapter 6 # 2, p 95.

3. From the text Chapter 6 # 3, p 95.

4. From the text Chapter 7 # 5, p 104.

5. From the text Chapter 7 # 6, p 104.

6. From the text Chapter 8 # 4, p 117. In addition please look back at my results in the notes with $n = 5$ and compute the exact distribution of the pivots. I suggest a graph of the probabilities of each of the 126 values of the statistic against the value. (Some values involve division by 0 and may be omitted from the graphs.)

7. From the text Chapter 8 # 5, p 117.

8. From the text Chapter 8 # 7, p 117.

Due date: 25 September 2012 in class.