

STAT 830

Problems: Assignment 3

1. From the text Chapter 8 # 4, p 117. In addition please look back at my results in the notes with $n = 5$ and compute the exact distribution of the pivots. I suggest a graph of the probabilities of each of the 126 values of the statistic against the value. (Some values involve division by 0 and may be omitted from the graphs.)
2. From the text Chapter 8 # 5, p 117.
3. From the text Chapter 8 # 7, p 117.
4. Suppose $\{X_{ij}; j = 1, \dots, n_i; i = 1, \dots, p\}$ are independent $N(\mu_i, \sigma^2)$ random variables. (This is the usual set-up for the one-way layout.)
 - (a) Find the MLE's for μ_i and σ .
 - (b) Find the expectations and variances of these estimators.
5. Let T_i be the error sum of squares in the i th cell in the previous question. In this question you may assume that

$$\frac{T_i}{\sigma^2} = \frac{\sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2}{\sigma^2}$$

has a χ^2 distribution with $n_i - 1$ degrees of freedom.

- (a) Find the best estimate of σ^2 of the form $\sum_1^p a_i T_i$ in the sense of mean squared error. That is, find the constants a_i which minimize the mean squared error.
 - (b) Do the same under the condition that the estimator must be unbiased.
6. In question 4 take $n_i = 2$ for all i and let $p \rightarrow \infty$. What is the limit of the MLE of σ ? Hint: find the mean and variance of the mle of σ^2 .

7. Suppose that Y_1, \dots, Y_n are independent random variables and that x_1, \dots, x_n are the corresponding values of some covariate. Suppose that the density of Y_i is

$$f(y_i) = \exp(-y_i \exp(-\alpha - \beta x_i) - \alpha - \beta x_i) 1(y_i > 0)$$

where α , and β are unknown parameters.

- (a) Find the log-likelihood, the score function and the Fisher information.
8. Consider the random effects one way layout. You have data $X_{ij}; i = 1, \dots, p; j = 1, \dots, n$ and a model $X_{ij} = \mu + \alpha_i + \epsilon_{ij}$ where the α 's are iid $N(0, \tau^2)$ and the ϵ 's are iid $N(0, \sigma^2)$. The α s are independent of the ϵ s.
- (a) Compute the mean and variance covariance matrix of the vector you get by writing out all the X_{ij} as a vector.
 - (b) Suppose that M is a matrix of the form $aI + b11^t$ where I is a $p \times p$ identity and 1 denotes a column vector of p ones. Show that M^{-1} is of the form $cI + d11^t$ and find c and d . In what follows you may use the fact that the determinant of M is $a^{p-1}(a + pb)$.
9. For each of the doses d_1, \dots, d_p a number of animals n_1, \dots, n_p are treated with the corresponding dose of some drug. The number dying at dose d is Binomial with parameter $h(d)$. A common model for $h(d)$ is $\log\{h/(1-h)\} = \alpha + \beta d$.
- (a) Find the likelihood equations for estimating α and β .
 - (b) Find the Fisher information matrix.
 - (c) Define the parameter LD50 as the value of d for which $h(d) = 1/2$; express LD50 as a function of α and β .

Due: Tuesday Oct 9 in class.