Lecture 11: Likelihood Asymptotics

- One form of Taylor's theorem: $f : \mathbb{R}^p \mapsto \mathbb{R}^q$ continuously differentiable on a ball containing x and y:

$$f(y) = f(x) + \int_0^1 Df(x + t(y-x))dt(y-x)$$

where Df is the derivative matrix of f, namely,

$$[Df(x)]_{ij} = \frac{\partial f_i(x)}{\partial x_j}.$$

- Apply to likelihood equations

$$U_n(\theta_0) = \int_0^1 V_n(\theta_0 + t(\hat{\theta}_n - \theta_0))dt(\hat{\theta}_n - \theta_0).$$

- I studied this equation carefully to prove

$$\mathcal{I}_1(\theta_0)\left[\sqrt{n}(\hat{\theta}_n - \theta_0)\right] - \frac{U_n(\theta_0)}{\sqrt{n}} \to 0$$

in probability.

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- Then Slutsky's theorem and the CLT prove

$$\mathcal{I}_1(\theta_0)\left[\sqrt{n}(\hat\theta_n - \theta_0)\right] \longrightarrow MVN(0, \mathcal{I}_1(\theta_))$$

or

$$\sqrt{n}(\hat\theta_n - \theta_0) \longrightarrow MVN(0, [\mathcal{I}_1(\theta_)]^{-1}) >$$

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- Then looked at 1 dimensional results. Fix $a \in \mathbb{R}^p$. Then

$$\frac{\sqrt{n}(a^T\hat\theta_n - a^T\theta_0}{\sqrt{a^T\mathcal{I}_1(\theta_0)]^{-1}a}} \Longrightarrow N(0,1),$$

$$\frac{\sqrt{n}(a^T\hat\theta_n - a^T\theta_0}{\sqrt{a^T\mathcal{I}_1(\hat\theta_n)]^{-1}a}} \Longrightarrow N(0,1),$$

$$\frac{\sqrt{n}(a^T\hat\theta_n - a^T\theta_0}{\sqrt{a^T[-V_n(\hat\theta_n)]^{-1}/n]a}} \Longrightarrow N(0,1),$$

$$\frac{\sqrt{n}(a^T\hat\theta_n - a^T\theta_0}{\sqrt{a^T[-V_n(\theta_0)]^{-1}/n]a}} \Longrightarrow N(0,1),$$

- And I introduced estimating equations.

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- Chapter 9.3-7, 9.10.
- Estimating equations are not really discussed in the text.
- In "course notes" on the web the MLE section from page 91 to 118 should be reviewed.