

Lecture 6: Statistical Functionals

- A statistical functional is a parameter which has a meaning for all or at least most cdfs F .
- Technically a statistical functional is a map $F \rightarrow T(F)$ which associates a cdf with a real number.
- Classic examples are mean, median, quantiles, other moments, characteristic functions, moment generating functions.
- Less obvious examples are things like

$$T(F) = \text{median}(\bar{X}_n)$$

the median of the distribution of \bar{X} for a sample of size n from F .

- A linear functional has

$$T(aF + bG) = aT(F) + bT(G).$$

- Example linear functionals are

$$T_r(F) = \int r(x)dF(x) = \int r(x)f(dx) = \mathbb{E}_F(r(X)).$$



Bootstrap and plug-in estimation

- The plug-in estimate of $T(F)$ is $T(\hat{F}_n)$.
- So the plug-in estimate of $T_r(F)$ is

$$\int r(x) d\hat{F}_n(x) = \frac{1}{n} \sum r(X_i).$$

- Notice how to do the integral: sum of values times probabilities because \hat{F}_n is a discrete distribution with n possible values and probability $1/n$ for each of those values.
- To compute more complicated plug in estimates we simulate.
- This is called *bootstrapping*.
- Generate samples of size n from \hat{F}_n and evaluate the object being studied for each sample. Average to get a Monte Carlo estimate of the \hat{F} expected value of the object.
- I bootstrapped the t pivot and $\bar{X} - \mu$.
- Then I studied the quality of resulting confidence intervals by Monte Carlo!



Course Coverage

- Chapter 7.2
- Chapter 8.1-3.

