STAT 870

*6pt

Problems: Assignment 1

1. Suppose A is a subset of Ω. What events are in the smallest σ-field containing A? [HINT: there aren't very many.]
2. Suppose X is a real valued function defined on Ω. Prove that if $\{X \le x\} \in \mathcal{F}$ for each real x then X is a real valued random variable on $(\Omega, \mathcal{F}, P)$. [Hint: Consider the family $\mathcal{G} = \{B : \{\omega : X(\omega) \in B\} \in \mathcal{F}\}$. Prove that $\mathcal{G}$ is a σ-field and then appeal to the definition of the Borel sigma-field. To use the definition of Borel in the notes remember that an open ball in R_1 is just an interval of the form (x_1, x_2).]
3. Suppose that A and B are independent events. Let $\mathcal{F}_A$ be the smallest σ-field containing the event A and similiarly defined $\mathcal{F}_B$. Show that $\mathcal{F}_A$ and $\mathcal{F}_B$ are independent.
4. Suppose $\{\mathcal{F}_i; i \in I\}$ is a family of σ-fields on Ω. Prove that $\mathcal{G} = \cap_i \mathcal{F}_i$ is a σ-field.
5. Show that for any family $\mathcal{C}$ of subsets of Ω there is a smallest σ-field on Ω containing $\mathcal{C}$.
6. Suppose X is an $\mathbf{R}^p$ valued random variable. Show that

   $$\mathcal{F}_X = \{X^{-1}(A) : A \in \mathcal{B}(\mathbf{R}^p)\}$$

   is a σ-field. This is the *σ-field generated by X*.
7. Show that $X_1, \ldots$ are independent if and only if $\mathcal{F}_{X_1}, \ldots, \mathcal{F}_{X_p}$ are independent.
8. From Text: page 19 # 32
9. From Text: page 19 # 35
10. From Text: page 20 # 46
11. From Text: page 85 # 27
12. From Text: page 85 # 28
13. From Text: page 85 # 29
14. From Text: page 91 # 74
15. From Text: page 148 # 20
16. From Text: page 149 # 27
17. From Text: page 151 # 38 (#40 in 8th Edition)
18. From Text: page 156 # 65 (#68 in 8th Edition)