STAT 870

Problems: Assignment 2

1. Show that if $Y > 0$ has cdf G then $\mathrm{E}(Y) = \int_0^\infty \{1 - G(y)\}\, dy$. Please try to do general Y but if not you may assume that Y is integer valued or has a density g.

2. Consider two urns A and B containing a total of N balls. An experiment is performed in which a ball is selected at random (all selections equally likely) at time $t (t = 1, 2, \ldots)$ from the collection of all N balls. Then an urn is selected at random (A is chosen with probability p and B is chosen with probability $q = 1 - p$) and the ball previously drawn is place in this urn. The state of the system at each trial is represented by the number of balls in A.

   (a) Determine the transition matrix for this Markov chain.

   (b) With the same setup suppose that an urn is chosen with probability proportional to the number of balls in the urn at time t. One of the balls in that urn is chosen at random and moved to an urn chosen at random with the probability that the destination urn will be urn A being k/N where k is the number of balls in A. Determine the transition matrix for this Markov chain.

3. A coin is tossed until 2 successive heads appear. Find the expected number of tosses required.

4. Customers arrive at a facility and wait there until a total of K customers have accumulated. Upon the arrival of the Kth customer all customers are served instantaneously and the process repeats. Let $\xi_0, \xi_1, \ldots$ be the number of customers arriving in successive periods and assume that the ξ are independent Bernoulli(α) random variables. Let X_n be the number of customers waiting at time n. Then $\{X_n\}$ is a Markov chain with states $0, 1, \ldots, K - 1$. Write out the transition matrix when $K = 3$.

5. At the end of a month a large retail store classifies each receivable account in one of four classes: current, 30 to 60 days overdue, 60 to 90 days overdue or over 90 days overdue. Assume that an account is paid in full with probability 0.95, 0.50, 0.20 and 0.10 depending on its status last month. In the long run what fraction of accounts are over 90 days overdue?

6. Chapter 4 #10, p 227

7. Chapter 4 #14, p 228

8. Chapter 4 #16, p 228

9. Chapter 4 #18, p 228

10. Chapter 4 #22, p 229

11. Chapter 4 #33, p 231

12. Chapter 4 #40, p 232
13. Chapter 4 #43, p 233
14. Chapter 4 #44, p 233